M/M/1 Retrial Queuing System with Vacation Interruptions under ERLANG-K Service

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ABSTRACT
Consider a single server retrial queueing system in which customers arrive in a Poisson process with arrival rate **λ.** Let k be the number of phases in the service station. Let the service time follows an Erlang K-type distribution with service rate kμ for each phase. The server goes for vacation after exhaustively completing the service to the customers. This vacation rate follows an exponential distribution with parameter ***α.*** The concept of **vacation interruption** is introduced in this paper that is, the server comes from the vacation into normal working condition without completing his vacation period subject to some conditions. We assume that the services in all phases are independent and identical and only one customer at a time is in the service mechanism. If the server is **free** at the time of a primary call arrival, the arriving call begins to be served in Phase 1 immediately by the server then progresses through the remaining phases and must complete the last phase and leave the system before the next customer enters the first phase. If the server is **busy**, then the arriving customer goes to orbit and becomes a source of **repeated calls**. This pool of sources of repeated calls may be viewed as a sort of queue. Every such source produces a Poisson process of repeated calls with intensity **σ**. If an incoming repeated call finds the server free, it is served in the same manner and leaves the system after service, while the source which produced this repeated call disappears. Otherwise, the system state does not change. We assume that the access from orbit to the service facility is governed by the **classical retrial policy.** This model is solved using **Matrix geometric Technique**. Numerical study have been done for Analysis of Mean number of customers in the orbit (MNCO), Probability of server free ,busy and in vacation for various values of **λ , μ ,k , α, N_0** and **σ** in elaborate manner and also various particular cases of this model have been discussed.

Keywords
Single Server – Erlang k-type service –K phases – exhaustive vacation - vacation interruption – threshold value - matrix Geometric Method – classical retrial policy

1. INTRODUCTION
Queueing systems in which arriving customers who find all servers and waiting positions (if any) occupied may retry for service after a period of time are called **Retrial queues.** Retrial queues have been studied by **Artalejo (1995) , Falin (1990 , 1997),** widely used to model many problems in telephone switching systems, telecommunication networks, computer networks and computer systems. For literature we refer classified bibliography of **Artalejo (1999**). Because of the complexity of the retrial queueing models, analytic results are generally difficult to obtain. There are a great number of numerical and approximations methods available, in this paper we will place more emphasis on the solutions by **Matrix geometric method** introduced by **M. F Neuts (1981,1990).** For literature study of matrix geometric method we refer the Bibliographical guide to the analysis of retrial queues through matrix analytic technique by **Gomez-Corral (2006).** The main characteristic of a retrial queue is that a primary customer finds the service facility busy upon arrival immediately leaves the service area, but after some time later he **repeats his demand**. Between trials a customer is said to be in ***'orbit'.*** The evolution of such queues exhibits an alternating sequence of idle and busy periods and, in contrast to standard queues it is possible to have an idle server while the orbit is not empty. We assume that the access from the orbit to the service facility follows the exponential distribution with rate **nσ** which may depend on the current number n, (n ≥ 0) the number of customers in the orbit. That is, the probability of repeated attempt during the interval (t, t +Δt), given that there are n customers in the orbit at time t is **nσ Δt** which is called the **classical retrial rate policy.** The input flow of primary calls, interval between repetitions and service time in phases are mutually independent. Vacation policies have been extensively discussed by many researchers in queueing and retrial queueing models. In this paper we introduce this vacation interruption policy for retrial queueing system with Erlang k type service. Vacation interruption was discussed by **Jihong Li** and **Naishuo Tian (2007)** in single server working vacation in classical queueing models.

2. MODEL DESRIPTION
Consider a single server retrial queueing system in which customers arrive in a Poisson process with arrival rate **λ**. These customers are identified as primary calls**.** Let k be the number of phases in the service station. Assume that the service time has **Erlang-k type distribution** with service rate kμ for

each phase. The server goes for a compulsory exhaustive type vacation with **vacation interruptions Li and Naishuo Tian (2007)**. This vacation rate follows an exponential distribution with parameter α.

2.1 Description of the Vacation Interruptions

In this model the server has to go for compulsory exhaustive type vacation if there are no customers in the orbit and the server goes for vacation once again after completing atleast one service this type of vacation in queueing theory is called Single vacation with exhaustive service.

The concept of **vacation interruption** discussed by **Jihong Li and Naishuo Tian (2007)** in the working vacation under classical queueing models who describe it as,

"*we introduce a new policy: the server can come back from the vacation to the normal working level once some indices of the system, such as the number of customers, achieve a certain value in the vacation period. The server may come back from the vacation without completing the vacation. Such policy is called vacation interruption*."

The vacation interruption policy for Retrial Queuing system for this model is described as,

"once the threshold value N_o, the number of customers in the orbit is reached, the server's vacation will be interrupted and will be called into the system for service immediately and the server starts the service as soon as either there is a primary call or there is a repeated call entering the system".

3. MATRIX GEOMETRIC METHOD

Let N(t) be the random variable which represents the number of customers in the orbit at time t and H(t) be the random variable which represents the phase in which the customer is being served at time t . The random process is described as

{ < N(t) ,H(t) >/ N(t)=0,1,2,3,…; H(t)=0,1,2,3,...,k,k+1 }.

H(t) = 0 means the server is idle at time t

H(t) = i means the server is busy with the customer in the i^{th} phase for i = 1,2,3,…,k

H(t) = k+1 means then the server is in vacation

The possible state spaces are

{ (u , v) / u = 0,1,2,3,…,N_0 -1 ; v = 0,1,2,3,…,k+1 } U

{ (u , v) / u = N_0, N_0+1,N_0+2,… ; v = 0,1,2,3,…,k }

The infinitesimal generator matrix Q is given below

$$\begin{pmatrix} A_{00} & A_0 & O & O & O & \dots \\ A_{10} & A_{11} & A_0 & O & O & \dots \\ O & A_{21} & A_{22} & A_0 & O & \dots \\ O & O & A_{32} & A_{33} & A_0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

The Matrices **A_{00}, $A_{n\ n-1}$, $A_{n\ n}$, $A_{n\ n+1}$ and A_{MM}** are defined as below

If $S_1 = -(\lambda+k\mu)$

The matrix A_{00} a square matrix of order k+2 is described as

$$\begin{pmatrix} -\lambda & \lambda & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & S_1 & k\mu & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & S_1 & k\mu & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & S_1 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & S_1 & k\mu & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & S_1 & k\mu \\ \alpha & 0 & 0 & 0 & \dots & 0 & 0 & -(\lambda+\alpha) \end{pmatrix}$$

The matrix $A_{n\ n-1}$ a square matrix of order k+2 is described as

$A_{n\ n-1}$ (n =1,2,3,…,N_0-1)

$$\begin{pmatrix} 0 & n\sigma & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix}$$

The matrix A_{nn-1} a square matrix of order k+1is described as

$A_{n\,n-1}$ $(n = N_0, N_0+1, N_0+2, \ldots)$

$$\begin{pmatrix} 0 & n\sigma & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \end{pmatrix}$$

The matrix A_{nn} a square matrix of order k+2 is described as

$A_{n\,n}$ $(n = 1, 2, 3, \ldots, N_0-1)$

$$\begin{pmatrix} -(\lambda+n\sigma) & \lambda & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & S_1 & k\mu & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & S_1 & k\mu & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & S_1 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & S_1 & k\mu & 0 \\ k\mu & 0 & 0 & 0 & \ldots & 0 & S_1 & 0 \\ \alpha & 0 & 0 & 0 & \ldots & 0 & 0 & -(\lambda+\alpha) \end{pmatrix}$$

The matrix A_{nn} a square matrix of order k+1 is described as

$A_{n\,n}$ $(n = N_0, N_0+1, \ldots)$

$$\begin{pmatrix} -(\lambda+n\sigma) & \lambda & 0 & 0 & \ldots & 0 & 0 \\ 0 & S_1 & k\mu & 0 & \ldots & 0 & 0 \\ 0 & 0 & S_1 & k\mu & \ldots & 0 & 0 \\ 0 & 0 & 0 & S_1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & S_1 & k\mu \\ k\mu & 0 & 0 & 0 & \ldots & 0 & S_1 \end{pmatrix}$$

The matrix A_{nn+1} a square matrix of order k+2 is described as

$A_{n\,n+1}$ $(n = 1, 2, 3, \ldots, N_0-2)$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & \lambda & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \lambda & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \lambda & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & \lambda & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 \end{pmatrix}$$

The matrix A_{nn+1} a square matrix of order k+2 is described as

$A_{n\,n+1}$ for $(n = N_0-1)$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & \lambda & 0 \\ \lambda & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \end{pmatrix}$$

The matrix A_{nn+1} a square matrix of order k+1is described as

$A_{n\,n+1}$ $(n = N_0, N_0+1, \ldots)$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & \lambda & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \lambda & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \lambda & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & \lambda & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & \lambda \end{pmatrix}$$

If the capacity of the orbit is finite say **M ($M > N_0$)**

The matrix A_{MM} a square matrix of order k+2 is described as

A_{MM}

$$\begin{pmatrix} -(\lambda+M\sigma) & \lambda & 0 & 0 & \ldots & 0 & 0 \\ 0 & -k\mu & k\mu & 0 & \ldots & 0 & 0 \\ 0 & 0 & -k\mu & k\mu & \ldots & 0 & 0 \\ 0 & 0 & 0 & -k\mu & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & -k\mu & k\mu \\ k\mu & 0 & 0 & 0 & \ldots & 0 & -k\mu \end{pmatrix}$$

Let **x** be a steady-state probability vector of Q and partitioned as

x = (x(0),x(1),x(2), ….) and **x** satisfies

$$\mathbf{XQ} = \mathbf{0}, \quad \mathbf{Xe} = \mathbf{1} \qquad (1)$$

Where

$\mathbf{x(i)} = (P_{i0}, P_{i1}, P_{i2}, P_{i3}, \ldots, P_{ik}, P_{ik+1}) \; (i=0,1,2,\ldots,N_0-1)$

$\mathbf{x(i)} = (P_{i0}, P_{i1}, P_{i2}, P_{i3}, \ldots, P_{ik}) \; (i \geq N_0)$

4. DIRECT TRUNCATION METHOD

In this method one can truncate the system of equations in **(1)** for sufficiently large value of the number of customers in the orbit, say M. That is, the orbit size is restricted to M such that any arriving customer finding the orbit full is considered lost. The value of M can be chosen so that the loss probability is small. Due to the intrinsic nature of the system in **(1)** the only choice available for studying M is through algorithmic methods. While a number of approaches is available for determining the cut-off point, M , The one that seems to perform well (with respect to approximating the system performance measures) is to increase M until the largest individual change in the elements of **X** for successive values is less than **ε** a predetermined infinitesimal value.

5. ANALYSIS OF STEADY STATE PROBABILITIES

In this paper we are applying the **Direct Truncation Method** to find the Steady state probability vector **X**. Let M denote the cut-off point for this truncation method. The steady state probability vector $\mathbf{X}^{(M)}$ is now partitioned as

$\mathbf{X}^{(M)} = (x(0), x(1), x(2), \ldots x(M))$

which satisfies

$\mathbf{X}^{(M)}\mathbf{Q} = \mathbf{0}, \; \mathbf{X}^{(M)}\mathbf{e} = \mathbf{1},$

where $\mathbf{x(i)} = (P_{i0}, P_{i1}, P_{i2}, P_{i3}, \ldots, P_{ik}, P_{ik+1}) \; (i=0,1,2,\ldots,N_0-1)$

$\mathbf{x(i)} = (P_{i0}, P_{i1}, P_{i2}, P_{i3}, \ldots, P_{ik}) \; (i = N_0, N_0+1, \ldots, M)$

The above system of equations are solved by exploiting the special structure of the co-efficient matrix. It is solved by GAUSS-JORDAN elementary transformation method. Since there is no clear cut choice for M, we may start the iterative process by taking, say M=1 and increase it until the individual elements of **x** do not change significantly. That is, if M^* denotes the truncation point then $\|\mathbf{x}^{M^*}(\mathbf{i}) - \mathbf{x}^{M^*-1}(\mathbf{i})\|_\infty < \epsilon$,where ϵ is an infinitesimal quantity.

6. STABILITY CONDITION

The inequality $\left(\frac{\lambda}{\mu}\right) < 1$ is the necessary and sufficient condition for system to be stable.

7. SYSTEM PERFORMANCE MEASURES

In this section some important performance measures along with formulas and their qualitative behaviour for this queueing model are studied. Numerical study has been dealt in very large scale to study these measures. Defining

P(n , 0) = Probability that there are n customers in the orbit and server is free

P(n , i) = Probability that there are n customers in the orbit, server is busy with customer in the i^{th} phase for i = 1,2,3,…k.

P(n , k+1) = Probability that there are n customers in the orbit and server is in vacation.

We can find various probabilities for various values of λ, μ, k, α, N_0 and σ and the following parameters can be easily studied with these probabilities

a. The probability mass function of Server state

Let H(t) be the random variable which represents the phase in which customer is getting service at time t .

H	P
0	$\sum_{i=0}^{\infty} p(i,0)$
j	$\sum_{i=0}^{\infty} p(i,j)$

for j = 1,2,3,…,k+1

b. The probability mass function number of customers in the orbit

Let X(t) be the random variable representing the number of customers in the orbit.

Prob (No customer in the orbit) = $\sum_{j=0}^{k+1} p(0, j)$

Prob (i customers in the orbit) = $\sum_{j=0}^{k+1} p(i, j)$

c. The Mean number of customers in the orbit

$$\text{MNCO} = \sum_{i=0}^{\infty} i\left(\sum_{j=0}^{k+1} p(i, j)\right)$$

d. The probability that the orbiting customer is blocked

$$\text{Blocking Probability} = \sum_{i=1}^{\infty}\sum_{j=1}^{k+1} p(i, j)$$

e. The probability that an arriving customer enter into service immediately

$$\text{PSI} = \sum_{i=0}^{\infty} p(i,0)$$

8. NUMERICAL STUDY

MNCO : Mean number of customers in the orbit

P_0 : Probability that the server is idle.

P_1 : Probability that the server is busy.

P_2 : Probability that the server is in vacation.

Table I and Table II show the impact of retrial rate over the system. Mean number of customers in the orbit decreases as σ increases.

Table III and Table IV show the effect of Threshold value N_0 over the system. As the threshold value increases, mean number of customers in the orbit increases. However the mean number of customers in the orbit is independent of the threshold value after N_0 reaches a certain limit after which, this model becomes a exhaustive type vacation model without interruptions.

Table I: Mean number of customers in the orbit for $\lambda = 4$, $\mu = 10$, $k = 5$, $\alpha = 1$, $N_0 = 5$ and various values of σ

σ	MNCO	P_0	P_1	P_2
10	1.7625	0.1917	0.4000	0.4083
20	1.6897	0.1248	0.4000	0.4752
30	1.6651	0.0991	0.4000	0.5009
40	1.6526	0.0855	0.4000	0.5145
50	1.6451	0.0771	0.4000	0.5229
60	1.6401	0.0714	0.4000	0.5286
70	1.6365	0.0673	0.4000	0.5327
80	1.6339	0.0642	0.4000	0.5358
90	1.6318	0.0617	0.4000	0.5383
100	1.6301	0.0598	0.4000	0.5402
200	1.6225	0.0508	0.4000	0.5492
300	1.6200	0.0477	0.4000	0.5523
400	1.6187	0.0462	0.4000	0.5538
500	1.6179	0.0452	0.4000	0.5548
600	1.6174	0.0446	0.4000	0.5554
700	1.6170	0.0442	0.4000	0.5558
800	1.6168	0.0439	0.4000	0.5561
900	1.6166	0.0436	0.4000	0.5564
1000	1.6164	0.0434	0.4000	0.5566
2000	1.6156	0.0425	0.4000	0.5575
3000	1.6154	0.0422	0.4000	0.5578
4000	1.6152	0.0420	0.4000	0.5580
5000	1.6152	0.0419	0.4000	0.5581
6000	1.6151	0.0418	0.4000	0.5582
7000	1.6151	0.0418	0.4000	0.5582
8000	1.6151	0.0418	0.4000	0.5582
9000	1.6150	0.0417	0.4000	0.5583
10000	1.6150	0.0417	0.4000	0.5583

Table II: Mean number of customers in the orbit for $\lambda = 8$, $\mu = 10$, $k = 5$, $\alpha = 1$, $N_0 = 5$ and various values of σ

σ	MNCO	P_0	P_1	P_2
10	5.7567	0.1504	0.8000	0.0496
20	4.5912	0.1011	0.8000	0.0989
30	4.2466	0.0760	0.8000	0.1240
40	4.0835	0.0613	0.8000	0.1387
50	3.9886	0.0516	0.8000	0.1484
60	3.9266	0.0448	0.8000	0.1552
70	3.8830	0.0398	0.8000	0.1602
80	3.8506	0.0359	0.8000	0.1641
90	3.8256	0.0329	0.8000	0.1671
100	3.8058	0.0303	0.8000	0.1697
200	3.7179	0.0186	0.8000	0.1814
300	3.6891	0.0145	0.8000	0.1855
400	3.6748	0.0124	0.8000	0.1876
500	3.6663	0.0112	0.8000	0.1888
600	3.6606	0.0103	0.8000	0.1897
700	3.6566	0.0097	0.8000	0.1903
800	3.6536	0.0093	0.8000	0.1907
900	3.6512	0.0089	0.8000	0.1911
1000	3.6493	0.0086	0.8000	0.1914
2000	3.6408	0.0073	0.8000	0.1927
3000	3.6380	0.0069	0.8000	0.1931
4000	3.6366	0.0067	0.8000	0.1933
5000	3.6358	0.0066	0.8000	0.1934
6000	3.6352	0.0065	0.8000	0.1935
7000	3.6348	0.0064	0.8000	0.1936
8000	3.6345	0.0064	0.8000	0.1936
9000	3.6343	0.0063	0.8000	0.1937

Table III: Mean number of customers in the orbit for $\lambda = 4$, $\mu = 10$, $k = 5$, $\alpha = 1$, $\sigma = 10$ and various values of N_0

Threshold	MNCO	P_0	P_1	P_2
5	1.7625	0.1917	0.4000	0.4083
15	3.2759	0.1563	0.4000	0.4437
25	3.6340	0.1530	0.4000	0.4470
35	3.6967	0.1526	0.4000	0.4474
45	3.7061	0.1526	0.4000	0.4474

55	3.7074	0.1526	0.4000	0.4474
65	3.7075	0.1526	0.4000	0.4474
75	3.7076	0.1526	0.4000	0.4474
85	3.7076	0.1526	0.4000	0.4474
95	3.7076	0.1526	0.4000	0.4474
105	3.7076	0.1526	0.4000	0.4474
115	3.7076	0.1526	0.4000	0.4474
125	3.7076	0.1526	0.4000	0.4474
135	3.7076	0.1526	0.4000	0.4474
145	3.7076	0.1526	0.4000	0.4474
155	3.7076	0.1526	0.4000	0.4474
165	3.7076	0.1526	0.4000	0.4474
175	3.7076	0.1526	0.4000	0.4474
185	3.7076	0.1526	0.4000	0.4474
195	3.7076	0.1526	0.4000	0.4474
205	3.7076	0.1526	0.4000	0.4474
215	3.7076	0.1526	0.4000	0.4474

Table IV: Mean number of customers in the orbit for $\lambda = 8$, $\mu = 10$, $k = 5$, $\alpha = 1$, $\sigma = 10$ and various values of N_0

Threshold	MNCO	P_0	P_1	P_2
5	5.7567	0.1504	0.8000	0.0496
15	7.1123	0.1302	0.8000	0.0698
25	7.9062	0.1248	0.8000	0.0752
35	8.2783	0.1232	0.8000	0.0768
45	8.4345	0.1227	0.8000	0.0773
55	8.4958	0.1225	0.8000	0.0775
65	8.5188	0.1225	0.8000	0.0775
75	8.5272	0.1225	0.8000	0.0775
85	8.5301	0.1225	0.8000	0.0775
95	8.5312	0.1225	0.8000	0.0775
105	8.5315	0.1225	0.8000	0.0775
115	8.5317	0.1225	0.8000	0.0775
125	8.5317	0.1225	0.8000	0.0775
135	8.5317	0.1225	0.8000	0.0775
145	8.5317	0.1225	0.8000	0.0775
155	8.5317	0.1225	0.8000	0.0775
165	8.5317	0.1225	0.8000	0.0775
175	8.5317	0.1225	0.8000	0.0775
185	8.5317	0.1225	0.8000	0.0775
195	8.5317	0.1225	0.8000	0.0775
205	8.5317	0.1225	0.8000	0.0775
215	8.5317	0.1225	0.8000	0.0775

9. CONCLUSIONS

If the server follows the Exhaustive type vacation policy with vacation interruption then it is observed from numerical studies that Mean number of customers in the orbit decreases as the retrial rate increases. If N_o, the threshold value increases, mean number of customers in the orbit increases. However the mean number of customers in the orbit is independent of the threshold value after N_0 reaches a certain limit after which, this model becomes a exhaustive type vacation model without interruptions. This research work can further be extended by introducing various parameters like negative arrival, second optional service etc.,

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