Edge-odd Gracefulness of $C_3 \Theta P_n$ and $C_3 \Theta 2p_n$

Dr. A. Solairaju
Associate Professor of Mathematics
Jamal Mohamed College, Tiruchirapalli – 620 020.
Tamil Nadu, India.

C. Vimala and A.Sasikala
Assistant Professors (SG),
Department of Mathematics,
Periyar Maniammai University, Vallam
Thanjavur – Post.. Tamil Nadu, India.

ABSTRACT
A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the Edge-odd gracefulness of $C_3 \Theta P_n$ and $C_3 \Theta 2P_n$ is obtained.

Key words: Graceful Graph, Edge-odd graceful labeling, Edge-odd graceful graph

1. INTRODUCTION
A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A.Solairaju, C.Vimala, A.Sasikala [2008] gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n, A. Solairaju et.al. [2010] that the cartesian product of path P_2 and circuit C_n for all integer n, $S_{m,n}$, $P_m \Theta S_5$ and $C_m \Theta S_n$ for n is even, are is edge-odd graceful. Here the edge-odd graceful labeling of $C_3 \Theta P_n$ and $C_3 \Theta 2P_n$ is obtained.

Section 2: Basic Concept
In this section, the following definitions are first listed.

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1,2, …, m} such that when each edge uv is assigned the label $|f(u) - f(v)|$ the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge – odd graceful graph [6]: A (p, q) connected graph is edge-odd graceful if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd.

Section 3: Edge-odd Gracefulness of Armed crown graph $C_3 \Theta P_n$

In this section edge-odd gracefulness of $C_3 \Theta P_n$ is obtained.

Definition 3.1: Armed crown $\mathbf{C_3} \Theta P_n$ is a connected graph obtained from the circuit C_3 by adding a path P_n to each vertex of C_3. It has 3n vertices and 3n edges. This graph is given in figure 1.

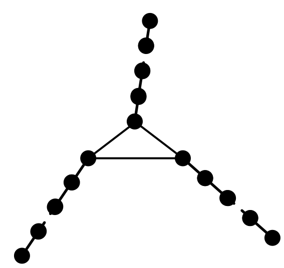

Figure 1: Graph of $C_3 \Theta P_n$

Lemma 3.2: The connected graph $C_3 \Theta P_2$ is edge – odd graceful.

The figure 2 is the armed crown $C_3 \Theta P_2$ with 6 vertices and 6 edges, with some arbitrary edge-odd graceful labeling to vertices and edges.

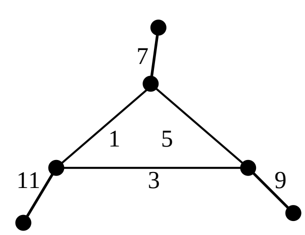

Figure 2: Graph of $C_3 \Theta P_2$

Lemma 3.3: The connected graph $C_3 \Theta P_3$ is edge – odd graceful.

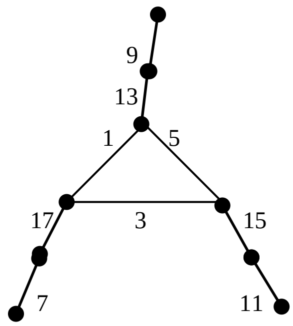

Figure 3: Graph of $C_3 \Theta P_3$

The graph in figure 3 is the armed crown $C_3 \Theta P_3$ with 9 vertices and 9 edges, with some arbitrary edge-odd graceful labeling to vertices and edges.

Lemma 3.4: The connected graph $C_3 \Theta P_4$ is edge – odd graceful.

The armed crown $C_3 \Theta P_4$ with 12 vertices and 12 edges is given in figure 4 with some arbitrary edge-odd graceful labeling to vertices and edges.

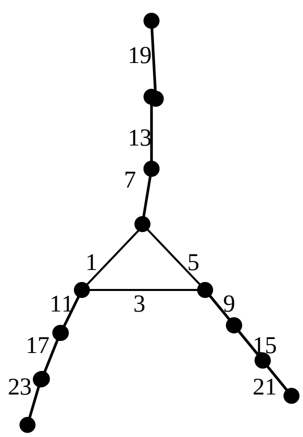

Figure 4: Graph of $C_3 \Theta P_4$

Theorem 3.5: The connected graph $C_3 \Theta P_n$ is edge – odd graceful for n > 4.

Proof:

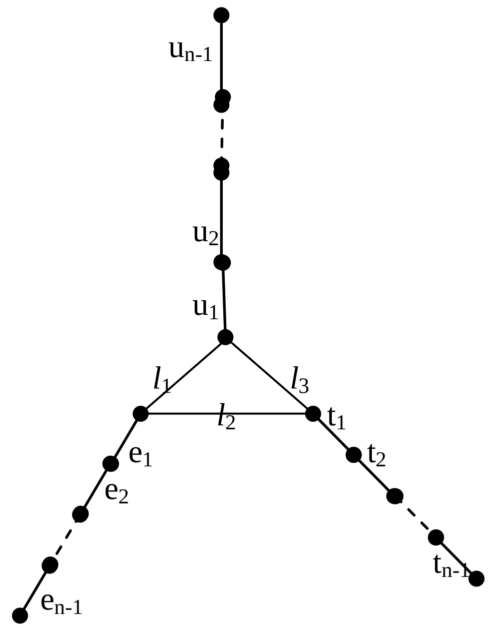

Figure 5: Graph of $C_3 \Theta P_n$

The figure 5 is the armed crown $C_3 \Theta P_n$ with 3n vertices and 3n edges, with some arbitrary labelings to its vertices and edges.

Define f: $E(G) \rightarrow \{1, 3, \ldots, 2q-1\}$ by
For n is even

$f(\ell i) = 2i - 1$, i = 1, 2, 3; $f(e_1) = q + 5$
$f(e_i) = f(e_1) + 3i - 3$, for i = 3, 5, …, (n-1)
$f(e_2) = 7$
$f(e_i) = f(e_2) + 3i$, for i = 4, 6, 8, …, (n-2);
$f(t_1) = f(e_1) - 2$
$f(t_i) = f(t_1) + 3i - 3$, for i = 3, 5, …, (n-1);
$f(t_2) = 11$
$f(t_i) = f(t_2) + 3i$, for i = 4, 6, 8, …, (n-2);
$f(u_1) = f(e_1) - 4$
$f(u_i) = f(u_1) + 3i - 3$, for i = 3, 5, …, (n-1); $f(u_2) = 9$
$f(u_i) = f(u_2) + 3i$, for i = 4, 6, 8, …, (n-2)

For n is odd

$f(\ell i) = 2i - 1$, i = 1, 2, 3; $f(e_1) = 11$
$f(e_i) = f(e_1) + 6i - 6$, i = 2, 3, …, (n-1) ;
$f(t_1) = 9$
$f(t_i) = f(t_1) + 6i - 6$, i = 2, 3, …, (n-1);
$f(u_1) = 7$
$f(u_i) = f(u_1)+6i-6$, i = 2, 3 ,…, (n-1) …. (1).

Define f_+: $V(G) \rightarrow \{0, 1, 2, \ldots, (2k-1)\}$ by $f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v …. (2).

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labeling for vertex set have distinct values in {1, 2,….,(2k-1)}. Hence the graph $C_3 \Theta P_n$ is edge-odd graceful.

Section 4 - Bi-armed crown C_3 Θ $2P_n$ is edge-odd graceful

Definition 4.1: Bi-armed crown $\mathbf{C}_3$ Θ $2P_n$ is a connected graph obtained from the circuit C_3 by adding two paths P_n to each vertex of C_3. It has (6n - 3) vertices and (6n - 3) edges.

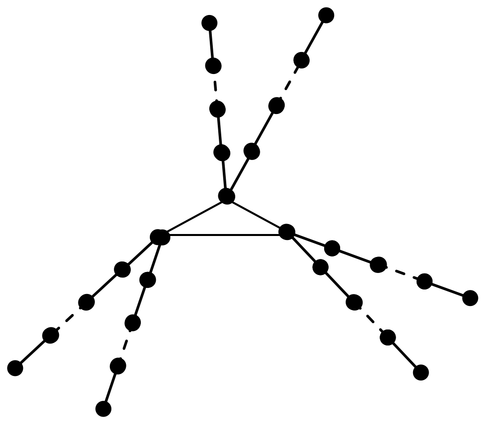

Figure 6: Graph of C_3 Θ $2P_n$

Theorem 4.2: The connected graph C_3 Θ $2P_n$ is edge – odd graceful.

Proof: The figure 7 is the armed crown C_3 Θ P_n with (6n - 3) vertices and (6n - 3) edges, with some arbitrary edge-odd graceful labeling to vertices and edges paths P_n to each vertex of C_3. It has (6n - 3) vertices and (6n - 3) edges.

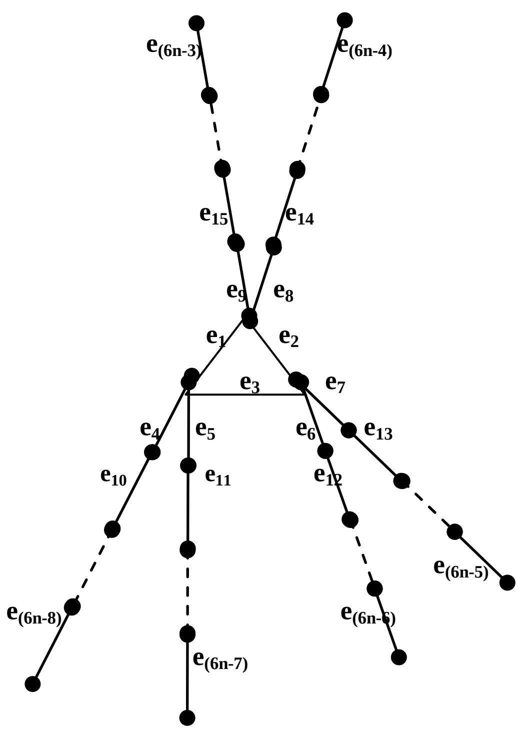

Figure 7: Graph of C_3 Θ $2P_n$

Define f: E(G) → {1, 3, …, 2q-1} by

For n is even

$f(ei) = 2i – 1, i = 1, 2, 3, …, (6n-3)$. (1)

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by

$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v (2)

Hence the map f and the induced map f_+ provide labels as odd numbers for edges with all distinct and also the labelings for vertex set has distinct values in {1, 2,…., (2k-1)}. Hence the graph C_3 Θ $2P_n$ is edge-odd graceful.

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