Application of Trace on Triangular Fuzzy Number Matrices

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ABSTRACT
The fuzzy matrices play an important role in various fields. In this research paper we discussed some elementary operations on proposed triangular fuzzy numbers (TFNs). We also defined some fuzzy arithmetic operations on triangular fuzzy matrices (TFMs). A description on trace of triangular fuzzy matrices (TTFMs) is proposed. Some more special properties of trace of triangular fuzzy matrices have also been discussed. Some applications of Numerical example are also provided here.

Keywords
Fuzzy Arithmetic, Fuzzy number, Triangular fuzzy number (TFN), Triangular fuzzy matrix(TFM),Trace of Triangular fuzzy matrix(TTFM).

1. INTRODUCTION
The concept of fuzzy set was introduced by Zadeh in 1965 [14].Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1].It can be used in a wide range of domains where information is incomplete and imprecise. In 1951 interval arithmetic was first suggested by Dwyer [4], by means of Zadeh's extension principle [15],the usual arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers. Dubosis and Prade [2]has defined many of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single-valued numbers. Jhon [6]studied an appraisal of theory an applications on fuzzy set of type-2.

We introduce triangular fuzzy matrices (TFM).To the best of our knowledge, no work is available on TFMs, though a lot of work on fuzzy matrices is available in literature. A brief review on fuzzy matrices is given below.

Fuzzy matrix has been proposed to represent fuzzy relation in a system based on fuzzy set theory [10].Fuzzy matrices were introduced for the first time by Thomason who discussed the convergence of power of fuzzy matrix. Several authors had presented a number of results on the convergence of power sequences of fuzzy matrices. Several authors have presented a number of results on the convergence of power sequence of fuzzy matrices [5,7] Fuzzy matrices play an important role in scientific development .Tow new operations and some applications of fuzzy matrices are given in [11,12,13]. Some important results on determinant of square fuzzy matrices presented by Kin [9] He defined the determinant of square fuzzy matrix and contributed with very research works [8]

. In this paper section 2 deal with some basic definition of triangular fuzzy number and operations on triangular fuzzy numbers (TFNs). In section 3, we have reviewed the definition of triangular fuzzy matrix (TFM) and some operations on triangular fuzzy matrices (TFMs). In section 4, we provide the trace of triangular fuzzy matrix(TTFM). In section 5,we have presented some properties of trace of triangular fuzzy matrix(TTFM).In section 6, an application of this operation are discussed and numerical example is given .Concluding remarks are given in section 7.

2. BASIC CONCEPT

2.1 Definition [6]
Let A be a classical set μ_A (x) be a real valued function defined from R→ $[0,1]$.A fuzzy setA^* with the function $\mu_A(x)$ is defined by $A^* = [(x, \mu_A(x)); x \in A$ and $\mu_A(x) \in [0,1]\}$.The function μ_A (x) is known as the membership function of A^*.

2.2 Definition
A fuzzy set A on R must possess at least the following three properties to verify as a fuzzy number,

(i) A must be a normal fuzzy set;
(ii) α_A must be closed interval for every $\alpha \in [0,1]$
(iii) The support of A, 0_A,must be bounded.

2.3 Triangular Fuzzy Number(TFN)[3]
It is a fuzzy number represented with three vertices as follows: A=(a, b, c) this representation is interpreted as membership functions and hold the following conditions

(i) a to b is strictly increasing function.

(ii) b to c is strictly decreasing function.

(iii) $a \leq b \leq c$

$$\mu_A(x) = \begin{cases} 0 & for\, x < a \\ \frac{x-a}{b-a} & for\, a \leq x \leq b \\ \frac{c-x}{c-b} & for\, c \leq x \leq d \\ 0 & forx > c \end{cases}$$

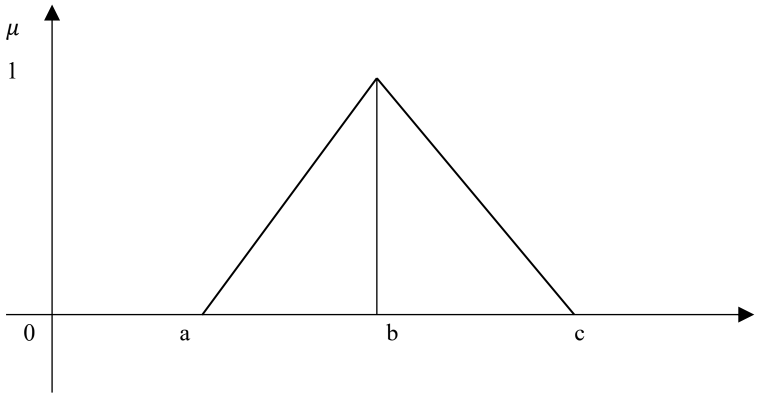

Figure 1.Triangular Fuzzy Number A=(a,b,c)

2.4 Operation of Triangular Fuzzy Number using Function Principle [1]

The following are the four operations that can be performed on TFN: Let A $= (a_1, a_2, a_3)$ and B $= (b_1, b_2, b_3)$ then,

(i) Addition: A(+)B= (a_1, a_2, a_3) (+)(b_1, b_2, b_3)

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

(ii) Subtraction : A(-)B= (a_1, a_2, a_3) (-)(b_1, b_2, b_3)

$= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

(iii) Multiplication: A(x) B= (a_1, a_2, a_3) (x)(b_1, b_2, b_3)

=(min($a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2,$ max($a_1b_1, a_1b_3, a_3b_1, a_3b_3$))

(iv) Scalar multiplication

$$\rho A = \begin{cases} (\rho a_1, \rho a_2, \rho a_3) if \rho \geq 0 \\ (\rho a_3, \rho a_2, \rho a_1) if\ \rho < 0 \end{cases}$$

Where ρ is scalar multiplication.

3. TRIANGULAR FUZZY MATRICES (TFMs)

In this section ,we discussed the Triangular fuzzy matrix and their operation of the matrices.

3.1 Definition

A triangular fuzzy matrix of order m x n is defined as A=$(a_{ij})_{mxn}$ where $a_{ij} = (a_{ij1}, a_{ij2}, a_{ij3})$ is the i^{th} rows and j^{th} columns element of A.

3.2 Arithmetic operations of Triangular Fuzzy Matrices (TFMs)

For algebra of matrices, we defineded the following operations of matrices on triangular fuzzy matrices. Let us consider A=(a_{ij}) nxn and B=(b_{ij})nxn be two triangular fuzzy matrices (TFMs) of same order. Then the following operations hold.

(i) Addition:

A(+)B=$(a_{ij} + b_{ij})$

(ii) Subtraction

A(-)B=$(a_{ij} - b_{ij})$

(iii) Multiplication

For A=(a_{ij}) mxn and B=(b_{ij})nxk then AB=(c_{ij})mxk where c_{ij}=$\sum_{h=1}^{n} a_{ih}b_{hj}$,i=1,2,…m and j=1,2,3,…k.

(iv)Transpose matrix

$A^T or A^/$=(a_{ji})

(v)Scalar multiplication

σA=(σa_{ij}) where σ is scalar.

3.3 Equal TFM

Two TFM A=(a_{ij}) and B=(b_{ij}) are said to be equal if

(i) they are of the same size and

(ii) the elements in the corresponding places of the two matrices are the same. i.e $a_{ij} = b_{ij}$ for each pair of subscripts I and j. Also it is denoted by A=B.

3.4 Null or Zero TFM

A TFM is said to be a null TFM if all its entries are zero, i.e all elements are (0,0,0).This matrix is denoted by 0.

3.5 Triangular TFM

A square TFM is said to be a triangular if either (a_{ij}) = (0,0,0) for all $i > j$ or $(a_{ij}$)=(0,0,0) for all $i < j; i, j = 1,2, \dots . n$.

A triangular TFM A=(a_{ij}) is said to be upper triangular TFM when (a_{ij}) = (0,0,0) for all I$i > j$ and said to be a lower triangular TFM $(a_{ij}$)=(0,0,0) for all $i < j$.

4. TRACK OF TRIANGULAR FUZZY MATRIX (TTFM)

In this section , we discussed the new matrix namely trace in the fuzzy form.

4.1 Trace of a matrix

Let A be asqure matrix of order n. The sum of the elements of a lying along the principal diagonal is called the trace of A.i.e tr(A) =$\sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots . + a_{nn}$.

For Example

$$A = \begin{pmatrix} 1 & 4 & -3 \\ 2 & 8 & 3 \\ 5 & 0 & 8 \end{pmatrix}$$

tr (A)= $\sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots . + a_{nn}$

$=a_{11}+a_{22}+a_{33}$=1+8+8=17

tr (A) =17

4.2 Trace of TFM.

Let A = (a_{ij}^{TFM}) be a square triangular fuzzy matrix of order nxn denoted by tr(A) is the sum of the principal diagonal elements.

5. SPECIAL PROPERTIES OF TRACE OF TFM

In this section, we discussed the properties of TFM.

5.1 Properties of Trace of Triangular Fuzzy matrix (TTFM)

5.1.1 Property

Let A=(a_{ij}^{TFM}) be any square TFMs of order n then ,

tr(A)= tr(A^T)

Proof :

Let A = (a_{ij}^{TFM}) beTrFMs where $a_{ij}^{TFM}=(a_{ij1},a_{ij2},a_{ij3})$

Now tr(A) =$\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E,F,R_m)$ where $E=\sum_{i=1}^{n} a_{ii1}$,$F=\sum_{i=1}^{n} a_{ii2}, R_m=\sum_{i=1}^{n} a_{ii3},$

Then tr(A)= $\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E,F,R_m)$.

$\therefore tr(A)$ =(E,F,R_m)= $tr\ (A^T)$

Hence tr(A)= tr(A^T)

5.1.2 Property:

Let A=(a_{ij}^{TFM}) and $B=(b_{ij}^{TFM})$be any two square TFMs of order n then ,

tr(A)+ tr(B)= tr (A+B)

Proof:

Let A=(a_{ij}^{TFM}) and $B=(b_{ij}^{TFM})$be two square TFMs

where $a_{ij}^{TFM}=(a_{ij1},a_{ij2},a_{ij3})$ and $b_{ij}^{TFM}=(b_{ij1},b_{ij2},b_{ij3})$

Now tr(A) =$\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E,F,R_m)$ where $E=\sum_{i=1}^{n} a_{ii1}$,$F=\sum_{i=1}^{n} a_{ii2}, R_m=\sum_{i=1}^{n} a_{ii3},$

Similarly , tr(B) =$\sum_{i=1}^{n} b_{ii}^{TFM}$=$(M,L,Q_m)$ where $M=\sum_{i=1}^{n} b_{ii1}$,$L=\sum_{i=1}^{n} b_{ii2}, Q_m=\sum_{i=1}^{n} b_{ii3},$

$\therefore tr(A)+tr(B)=\sum_{i=1}^{n} a_{ii}^{TFM}+\sum_{i=1}^{n} b_{ii}^{TFM}$==$(E,F,R_m)+(M,L,Q_m)$

=$(E+M,F+L,R_m+Q_n)$

Again let A+B=D = (d_{ij}^{TFM}), where d_{ij}^{TFM} =$(a_{dij1},a_{dij2},a_{dij3})$,$a_{dij1=a_{ij1}+b_{ij1,}}$

$a_{dij2=a_{ij2}+b_{ij2,}}$, $a_{dij3=a_{ij3}+b_{ij3,}}$

Now $tr\ (D)=\sum_{i=1}^{n} d_{ij}^{TFM}$= $\sum_{i=1}^{n}(a_{ij1}+b_{ij1},a_{ij2}+b_{ij2},a_{ij3}+b_{ij3}\ )$

=$(E+M,F+L,R_m+Q_n,)$

Therefore, tr(A)+ tr(B)= tr (A+B)

5.1.3 Property

Let A=(a_{ij}^{TFM}) and $B=(b_{ij}^{TFM})$be any two square TFMs of order n then ,

tr(AB)= tr (BA)

Proof:

Let A=(a_{ij}^{TFM}) and $B=(b_{ij}^{TFM})$be two square TFMs

where $a_{ij}^{TFM}=(a_{ij1},a_{ij2},a_{ij3})$ and $b_{ij}^{TFM}=(b_{ij1},b_{ij2},b_{ij3})$

Also let C=AB=(c_{ij}^{TFM}) then c_{ij}^{TFM} =$\sum_{h=1}^{n} a_{ih}b_{hj}$

Now $tr(AB)=\sum_{i=1}^{n} c_{ii}^{TFM}$ =$\sum_{i=1}^{n}\sum_{h=1}^{n} a_{1h}b_{hi}$

Again let $BA=D=(d_{ij}^{TFM})$ where d_{ij}^{TFM} =$\sum_{h=1}^{n} b_{ih}a_{hj}$

Now $tr(BA)=\sum_{i=1}^{n}\sum_{h=1}^{n} b_{ih}a_{hi}$

=$\sum_{i=1}^{n}\sum_{h=1}^{n} a_{ih}b_{hi}$,interchanging the indices i and j.

Therefore tr (AB)= tr (BA).

5.1.4 Property

Let A=(a_{ij}^{TFM}) be any square TFMs of order n then ,

tr(kA)= k tr(A^T)

Proof :

Let A = (a_{ij}^{TFM}) beTFMs where $a_{ij}^{TFM}=(a_{ij1},a_{ij2},a_{ij3})$

Now tr(A) =$\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E,F,R_m)$ where $E=\sum_{i=1}^{n} a_{ii1}$,$F=\sum_{i=1}^{n} a_{ii2}, R_m=\sum_{i=1}^{n} a_{ii3},$

Then tr(kA)= $\sum_{i=1}^{n} ka_{ii}^{TFM}$=$(kE,kF,kR_m)$.

Where k $E=\sum_{i=1}^{n} ka_{ii1}$,$kF=\sum_{i=1}^{n} ka_{ii2}, kR_m=\sum_{i=1}^{n} ka_{ii3},$

Similarly, k tr(A) =$k\sum_{i=1}^{n} a_{ii}^{TFM}$=k$(E,F,R_m,)$ where $kE=k\sum_{i=1}^{n} a_{ii1}$,$kF=k\sum_{i=1}^{n} a_{ii2}, kR_m=k\sum_{i=1}^{n} a_{ii3}$

$\therefore (kE,KF,KR_m)=k(E,F,R_m)$

$\sum_{i=1}^{n} ka_{ii}^{TFM}=\ k\sum_{i=1}^{n} a_{ii}^{TFM}$

$\therefore tr(kA)=ktr(A)$

Hence $tr(kA)=ktr(A)$

5.1.5 Property:

The sum of two upper triangular TFMs of order n is upper triangular TFM.

Proof:

Let A=(a_{ij}^{TFM}) and $B=(b_{ij}^{TFM})$be two upper triangular square TFMs

where $a_{ij}^{TFM}=(a_{ij1},a_{ij2},a_{ij3})$ and $b_{ij}^{TFM}=(b_{ij1},b_{ij2},b_{ij3})$

Since A and B are upper triangular TFMs then a_{ij}^{TFM}=(0,0,0) and b_{ij}^{TFM}=(0,0,0) for all$i>j; i,j=1,2,\dots n$

let C=AB=(c_{ij}^{TFM})

where c_{ij}^{TFM} =$\sum_{h=1}^{n} a_{ih}b_{hj}$=$\sum_{h=1}^{n}(a_{ih1},a_{ih2},a_{ih3})\ (b_{hj1},b_{hj2},b_{hj3})$

Now we have to prove that c_{ij}^{TFM}=(0,0,0) if $i > j; i,j = 1,2,\dots,n$

For $i > j$ we have a_{ih}=(0,0,0) for k=1,2,…,i-1 and similarly b_{hj}=(0,0,0) for k=i,i+1,…,n.

Therefore c_{ij}^{TrFM} =$\sum_{h=1}^{n} a_{ih}b_{hj}$ =$\sum_{h=1}^{i-1} a_{ih}b_{hj} + \sum_{h=1}^{n} a_{ih}b_{hj}$

=(0,0,0)

Now c_{ii}^{TFM} =$\sum_{h=1}^{n} a_{ih}b_{hi}$ =$\sum_{h=1}^{i-1} a_{ih}b_{hi+a_{ii}b_{ii}} + \sum_{h=i+1}^{n} a_{ih}b_{hi}$

=(0,0,0)=$a_{ii}b_{ii}$

Since a_{ik} =(0,0,0) for h= 1,2,…,i-1 and b_{hi} =(0,0,0,) for h=i+1,i+2,…,n

Hence complete the prove

5.1.6 Property:

The sum of two lower triangular TFMs of order n is lower triangular TFM.

Proof:

Let A=(a_{ij}^{TFM}) and $B = (b_{ij}^{TFM})$be two lower triangular square TrFMs

where a_{ij}^{TFM}= $(a_{ij1}, a_{ij2}, a_{ij3})$ *and* b_{ij}^{TFM}= $(b_{ij1}, b_{ij2}, b_{ij3})$

Since A and B are lower triangular TFMs then a_{ij}^{TFM}=(0,0,0) and b_{ij}^{TFM}=(0,0,0) for all$i < j; i,j = 1,2,\dots n$

let C=AB=(c_{ij}^{TFM})

where c_{ij}^{TFM} =$\sum_{h=1}^{n} a_{ih}b_{hj}$=$\sum_{h=1}^{n}(a_{ih1}, a_{ih2}, a_{ih3})\,(b_{hj1}, b_{hj2}, b_{hj3})$

Now we have to prove that c_{ij}^{TFM}=(0,0,0) if $i < j; i,j = 1,2,\dots,n$

For $i < j$ we have a_{ih}=(0,0,0) for k=1,2,…,i-1 and similarly b_{hj}=(0,0,0) for k=i,i+1,…,n.

Therefore c_{ij}^{TFM} =$\sum_{h=1}^{n} a_{ih}b_{hj}$ =$\sum_{h=1}^{i-1} a_{ih}b_{hj} + \sum_{h=1}^{n} a_{ih}b_{hj}$

=(0,0,0)

Now c_{ii}^{TFM} =$\sum_{h=1}^{n} a_{ih}b_{hi}$ =$\sum_{h=1}^{i-1} a_{ih}b_{hi+a_{ii}b_{ii}} + \sum_{h=i+1}^{n} a_{ih}b_{hi}$

=(0,0,0)=$a_{ii}b_{ii}$

Since a_{ik} =(0,0,0) for h= 1,2,…,i-1 and b_{hi} =(0,0,0,) for h=i+1,i+2,…,n

Hence complete the prove.

6. NUMERICAL EXAMPLES

tr(A)= tr(A^T)

Solution:

Let us consider A=$\begin{pmatrix} (0,1,4) & (-2,0,2) & -3\ 2,7 \\ (2,4,6) & (-4\ 3,7) & (-1,0,4) \\ (-8,-6,12) & (0,2,4) & (-3,0,3) \end{pmatrix}$

L.S.H:

A=$\begin{pmatrix} (0,1,4) & (-2,0,2) & (-3\ 2,7) \\ (2,4,6) & (-4\ 3,7) & (-1,0,4) \\ (-8,-6,12) & (0,2,4) & (-3,0,3) \end{pmatrix}$

Now tr(A) =$\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E, F, R_m)$

= (-7,4,14)

and A^T= $\begin{pmatrix} (0,1,4) & (2,4,6) & (-8,-6,12) \\ (-2,0,2) & (-4,3,7) & (0,2,4) \\ (-3,2,7) & (-1,0,4) & (-3,0,3) \end{pmatrix}$

tr(A) =$\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E, F, R_m)$

=(-7,4,14)

Therefore tr(A)= tr(A^T).

Again,

tr(A)+ tr(B)= tr (A+B)

Solution:

Let us consider A=$\begin{pmatrix} (0,1,4) & (-2,0,2) & -3\ 2,7 \\ (2,4,6) & (-4\ 3,7) & (-1,0,4) \\ (-8,-6,12) & (0,2,4) & (-3,0,3) \end{pmatrix}$ and

B= $\begin{pmatrix} (0,1,4) & (-2,4,12) & (-3,3,10) \\ (2,4,5) & (-4,0,8) & (3,4,7) \\ (0,8,10) & (2,4,7) & (0,1,4) \end{pmatrix}$

L.S.H:

A=$\begin{pmatrix} (0,1,4) & (-2,0,2) & -3\ 2,7 \\ (2,4,6) & (-4\ 3,7) & (-1,0,4) \\ (-8,-6,12) & (0,2,4) & (-3,0,3) \end{pmatrix}$

Now tr(A) =$\sum_{i=1}^{n} a_{ii}^{TFM}$=$(E, F, R_m)$

= (-7,4,14) ⟶ (*)

and

B= $\begin{pmatrix} (0,1,4) & (-2,4,12) & (-3,3,10) \\ (2,4,5) & (-4,0,8) & (3,4,7) \\ (0,8,10) & (2,4,7) & (0,1,4) \end{pmatrix}$

tr(B) =$\sum_{i=1}^{n} b_{ii}^{TFM}$=$(M, L, Q_m)$ ⟶ (**)

=(-4,2,16)

Equation (*) and (**) we have

tr(A)+ tr(B)= (-7,4,14) +(-4,2,16) =(-11,6,30)

⟶ (***)

R.H.S :

A=$\begin{pmatrix} (0,1,4) & (-2,0,2) & -3\ 2,7 \\ (2,4,6) & (-4\ 3,7) & (-1,0,4) \\ (-8,-6,12) & (0,2,4) & (-3,0,3) \end{pmatrix}$ and

$$B=\begin{pmatrix}(0,1,4) & (-2,4,12) & (-3,3,10)\\ (2,4,5) & (-4,0,8) & (3,4,7)\\ (0,8,10) & (2,4,7) & (0,1,4)\end{pmatrix}$$

$$A+B=\begin{pmatrix}(0,1,4) & (-2,0,2) & (-3\ 2,7\\ (2,4,6) & (-4\ 3,7) & (-1,0,4)\\ (-8,-6,12) & (0,2,4) & (-3,0,3)\end{pmatrix}$$

$$+\begin{pmatrix}(0,1,4) & (-2,4,12) & (-3,3,10)\\ (2,4,5) & (-4,0,8) & (3,4,7)\\ (0,8,10) & (2,4,7) & (0,1,4)\end{pmatrix}$$

$$=\begin{pmatrix}(0,2,8) & (-4,4,14) & (-6,5,17)\\ (4,8,11) & (-8,3,15) & (2,4,11)\\ (-8,2,22) & (2,6,11) & (-3,1,7)\end{pmatrix}$$

Tr(A+ B)= (-11,6,30) ⟶ (****)

Form equation (***) and(****) we have

Tr(A)+tr(B)= tr(A+B)

7. CONCLUSION

In this paper, the main focus the description of trace of triangular fuzzy matrices are defined and also some special properties of trace of TFMs are proved. By using the trace of triangular fuzzy matrix work can be extended to another domain of diagonal of matrices. This also can be applied in different fuzzy number matrix.

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