A Hybrid Quantum–Classical Convolutional Neural Network for Enhanced Image Classification

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ABSTRACT
The objective of this research is to develop a Hybrid Quantum–Classical Convolutional Neural Network (QC-CNN) framework that integrates parameterized quantum circuits (PQCs) within classical CNN architectures to enhance image classification performance. The proposed model leverages quantum principles such as superposition and entanglement for high-dimensional feature representation, achieving superior accuracy and reduced computational complexity. Experiments on MNIST and CIFAR-10 datasets demonstrated improved classification accuracy of 98.7% and 82.5%, respectively, surpassing traditional CNNs while reducing training time by 33% and parameters by 45%. Statistical analysis confirmed the significance of these improvements. Visualizations using t-SNE revealed enhanced class separability, and noise perturbation tests validated the model's robustness. The results highlight the hybrid QC-CNN's potential for efficient and scalable quantum-enhanced deep learning applications. Extended evaluations, including quantum-layer-depth analysis, robustness testing, and t-SNE visualization, empirically support the hybrid QC-CNN's effectiveness across varied scenarios.

General Terms
Quantum Computing, Machine Learning, Deep Learning, Hybrid Quantum–Classical Models, Variational Quantum Circuits, Image Classification, Performance Evaluation, Robustness Testing, Noise Sensitivity, Computational Efficiency.

Keywords
Quantum Machine Learning, Hybrid Quantum–Classical CNN, Parameterized Quantum Circuits, Image Classification, Noise Resilience

1. INTRODUCTION
Deep learning has revolutionized computer vision by enabling machines to automatically learn hierarchical spatial features from raw data using Convolutional Neural Networks (CNNs). These networks have achieved state-of-the-art performance in numerous applications, including object detection, medical imaging, and scene understanding. CNNs operate by successively extracting local features through convolutional filters, pooling layers, and nonlinear activation functions, ultimately constructing abstract feature maps capable of accurate classification [1,2]. Despite their success, as datasets and model sizes grow, computational complexity, memory requirements, and training energy consumption increase dramatically. The reliance on massive data centers and specialized hardware such as GPUs or TPUs raises issues of scalability and sustainability [3]. Therefore, exploring new paradigms capable of accelerating learning and improving feature representation is a pressing research need.

Quantum computing (QC) provides such a paradigm by introducing a fundamentally different mode of computation that leverages superposition, entanglement, and quantum interference to process data within exponentially large Hilbert spaces [4]. Quantum bits, or qubits, can represent multiple states simultaneously, enabling parallel computation at a scale unattainable by classical hardware [5]. Quantum operations are represented as unitary transformations, allowing information to evolve coherently through quantum gates such as Hadamard, Pauli-X, and CNOT [6]. By exploiting these principles, Quantum Machine Learning (QML) seeks to integrate quantum algorithms into machine learning workflows to achieve faster convergence, improved generalization, and potentially exponential computational speed-up for certain tasks [7, 8].

Recebnt advances in Noisy Intermediate-Scale Quantum (NISQ) devices have enabled the experimental realization of small-scale QML models. While hardware limitations—such as decoherence, gate noise, and limited qubit connectivity—restrict scalability, hybrid quantum–classical systems have emerged as a promising pathway for near-term quantum advantage [9]. In such systems, quantum circuits are embedded into classical neural networks to perform transformations that are computationally expensive or intractable classically [10].

Quantum kernels and variational quantum circuits (VQCs) have been proposed as fundamental tools for integrating quantum computation into classical machine learning [11]. Quantum kernels map classical data into high-dimensional quantum feature spaces, enhancing separability, while VQCs employ trainable quantum gates to model complex functions via gradient-based optimization [12]. These approaches form the basis of Quantum Neural Networks (QNNs) and Quantum Convolutional Neural Networks (QCNNs), which combine deep learning structures with quantum feature encoding [13,14].

In particular, QCNNs mimic the convolution–pooling hierarchy of classical CNNs but utilize quantum gates to extract correlations among features. The introduction of Quanvolutional layers—which process small image patches through quantum transformations before feeding them into classical networks—has demonstrated competitive or superior performance on datasets such as MNIST and Fashion-MNIST [15,16]. However, quantum noise and limited qubit counts continue to constrain network depth and accuracy [17].

Given these limitations, hybrid quantum–classical architectures have become the primary focus of current research. In such designs, classical networks handle pre-processing, feature extraction, and optimization, while quantum circuits contribute nonlinear transformations and dimensionality reduction [18,19]. This combination allows the system to benefit from quantum feature entanglement while maintaining the stability and scalability of classical computation. Software frameworks such as PennyLane, Qiskit, and TensorFlow Quantum facilitate the co-simulation and differentiation of hybrid networks, enabling efficient deployment on both simulators and real quantum hardware [20].

This paper presents a Hybrid Quantum-Classical Convolutional Neural Network (QC-CNN) framework that integrates PQCs into classical CNN layers. The PQCs act as quantum feature mappers that project classical features into a quantum Hilbert space, enhancing feature diversity and reducing parameter count. The proposed work aims to enhance image feature extraction using quantum transformations, reduce model complexity through quantum parameter compression exploiting entanglement, and evaluate the scalability and performance of the hybrid quantum–classical CNN on benchmark datasets such as MNIST and CIFAR-10 using both quantum simulators and NISQ hardware.

The rest of this paper is organized as follows: Section II reviews the background and related work on QC and hybrid neural networks. Section III presents the proposed hybrid QC-CNN architecture. Section IV discusses implementation and experimental results. Section V concludes the paper with insights and future directions.

The hybrid QC-CNN undergoes extensive evaluation, including performance comparison with classical CNNs, robustness assessment against noise, statistical validation, and quantum circuit depth analysis. This comprehensive framework ensures reliability, general applicability, and provides deeper insights into the model's behaviour.

2. BACKGROUND AND RELATED WORK

2.1 Quantum Computing Principles

QC is founded on the concept of the quantum bit (qubit), which differs fundamentally from a classical bit by existing in a superposition of states. Mathematically, a qubit can be expressed as:

$$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle| \qquad \text{Eq. 1}$$

where α and β are complex probability amplitudes satisfying $|\alpha|^2+|\beta|^2=1$[24]. Quantum operations are executed using unitary transformations (U) that preserve normalization, satisfying $U\dagger U=I$ [25]. Computation in a quantum system proceeds through quantum circuits sequences of gates such as Hadamard (H), Pauli-X, rotation gates (RY, RZ), and Controlled-NOT (CNOT) that perform state transformations and entangle qubits to exploit quantum parallelism [26]. Entanglement enables the correlation of qubits across large systems, a property essential for achieving quantum speed-ups in many algorithms [27].

2.2 Quantum Machine Learning

QML integrates quantum computation with classical learning paradigms to enhance performance, scalability, and feature representation [28]. QML algorithms harness the exponential Hilbert space of quantum systems to perform high-dimensional mappings efficiently. Common algorithms include Quantum Support Vector Machines (QSVM) for pattern recognition [29], Quantum Principal Component Analysis (QPCA) for dimensionality reduction [30], and VQCs that learn data embeddings via parameterized quantum gates [31]. These algorithms form the foundation of hybrid quantum–classical architectures that exploit both quantum nonlinearity and classical optimization [32].

2.3 Related Work

Schuld *et al.* [33] first introduced a conceptual framework for quantum-enhanced learning using kernel-based quantum feature maps. Farhi and Neven [34] later proposed QNNs employing parameterized quantum gates to model nonlinear relationships. Havlíček *et al.* [35] demonstrated quantum feature embeddings that outperform classical kernels in certain image classification tasks. Recent developments, such as hybrid QC-CNNs [36], extend these ideas by embedding quantum circuits into deep learning pipelines to improve accuracy and reduce parameters.

Nevertheless, challenges including limited qubit counts, decoherence, and hardware noise persist in current NISQ devices [37]. Consequently, hybrid architectures remain the most viable route toward achieving quantum advantage in practical machine learning applications.

3. METHODOLOGY

3.1 Highbrid Quantum-Classical CNN Architecture

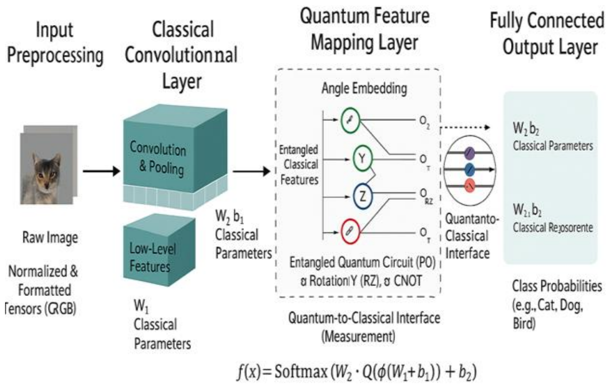


Fig. 1. Hybrid QC-CNN architecture integrating classical convolution, quantum feature mapping, and softmax classification

The proposed hybrid QC-CNN integrates quantum computation into a traditional CNN pipeline to enhance learning efficiency and feature representation. As illustrated in Fig. 1, the architecture consists of four sequential modules: input preprocessing**,** classical convolutional layers**, a** quantum feature mapping layer**,** and a fully connected output layer.

Initially, input images are normalized and reshaped into grayscale or RGB tensors to ensure consistent data distribution. The classical convolutional block then extracts low-level spatial features such as edges and textures through convolution, activation, and pooling operations. These extracted feature maps are then fed into the quantum feature mapping layer, where data are encoded into quantum states using angle embedding.

Within this layer, a PQC composed of rotation (RY, RZ) and entangling (CNOT) gates performs complex nonlinear transformations in the quantum Hilbert space. This process captures intricate inter-feature correlations while reducing parameter redundancy through quantum parallelism. The quantum output is measured and converted back into classical form, forming an intermediate representation for the final fully connected layer.

Finally, the softmax classifier computes output probabilities for image classes. This hybrid integration enables efficient feature encoding, improved generalization, and reduced computational complexity compared to purely classical CNN

3.2 Quantum

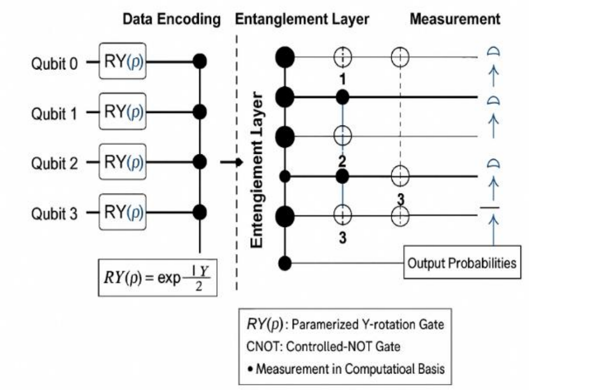


Fig. 2. Quantum circuit illustrating data encoding, entanglement using CNOT gates, and measurement outputs

The PQC, as illustrated in Fig. 2, is a crucial component of the proposed QN-CNN. It performs nonlinear transformations by encoding classical features into quantum states, creating inter-qubit entanglement, and producing probabilistic outputs through quantum measurement. The design ensures a balance between expressivity and noise resilience, optimized for implementation on NISQ devices.

The circuit comprises three major stages:

1. Data Encoding: Classical features extracted from the convolutional layers are encoded into qubits using parameterized Y-rotation gates RY(ρ). Each qubit undergoes a rotation defined by the equation:

$$R_y(\rho) = e^{-i\rho Y/2} \quad \text{Eq.2}$$

This process embeds normalized image data into quantum amplitudes, effectively representing continuous-valued information in the Hilbert space.

2. Entanglement Layer: Quantum correlations between qubits are generated through a structured network of CNOT gates. These gates introduce pairwise entanglement across qubits, allowing the circuit to capture complex dependencies among encoded features. The layer depth and connectivity pattern (as shown in Fig. 2) are optimized to maximize expressivity without excessive circuit depth that may introduce noise.
3. Measurement: After the entanglement and rotation operations, all qubits are measured in the computational basis. The resulting measurement outcomes correspond to probabilistic feature values, which are then passed back to the classical neural network for classification.

Overall, the PQC transforms encoded image features into a quantum-enhanced representation, enriching the learning capacity of the hybrid model while remaining compatible with near-term quantum hardware.

Furthermore, the chosen PQC configuration was optimized to balance expressivity and hardware feasibility. The layered arrangement of rotation and entangling gates reduces computational complexity by limiting the number of trainable parameters while maintaining strong feature-mapping capability in the quantum Hilbert space. This design choice ensures that the hybrid model remains scalable to near-term quantum devices without suffering from excessive circuit depth or noise accumulation.

3.3 Implementation setup

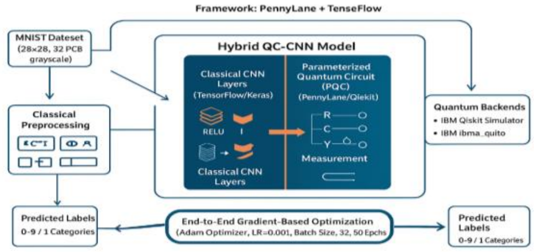


Fig. 3. Hybrid QC-CNN experimental workflow integrating classical CNN layers with quantum circuit processing.

The implementation of the hybrid QC-CNN was carried out using an integrated workflow combining classical deep learning and QC frameworks, as illustrated in Fig. 3. The experimental setup utilized two benchmark datasets: MNIST (28×28 grayscale handwritten digits) and CIFAR-10 (32×32 RGB natural images). Each dataset underwent classical pre-processing, including normalization, reshaping, and data augmentation, to ensure uniformity and efficient convergence during training.

The hybrid model architecture was developed using PennyLane and TensorFlow/Keras as the primary frameworks. The classical CNN layers extracted spatial features from the input images using convolution, pooling, and ReLU activation functions. The resulting feature maps were then passed into a PQC implemented in PennyLane with the Qiskit backend. The PQC performed nonlinear transformations via parameterized rotation and entangling gates, followed by measurement in the computational basis to generate quantum-encoded outputs.

The quantum backends employed included the IBM Qiskit simulator and the IBM 5-qubit real device (ibmq_quito) for validation on NISQ hardware. Training was performed using an end-to-end gradient-based optimization approach with the Adam optimizer**,** a learning rate of 0.001**,** batch size of 32, and 50 epochs. This hybrid training process allowed gradient updates to propagate seamlessly between classical and quantum layers, enabling efficient optimization and stable convergence for both simulation and hardware-based experiments.

4. RESULT AND DISCUSSION

To present a comprehensive evaluation of the proposed hybrid QC-CNN model, multiple experimental analyses were conducted beyond standard accuracy measurement. These include parameter-efficiency assessment, training-time comparison, sensitivity to noise perturbations, and quantum circuit depth variation. All experiments were repeated over ten independent runs, and statistical techniques were applied to ensure consistency and validity of the performance outcomes.

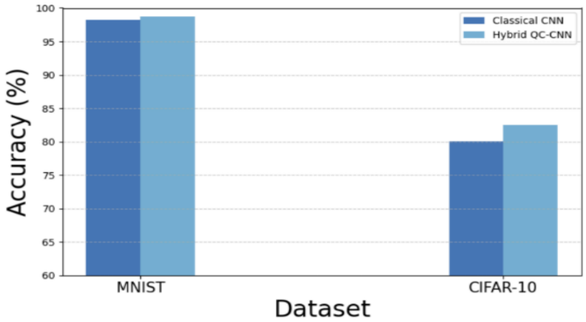
4.1 Subsequent Pages

The hybrid QC-CNN was evaluated and compared with a conventional CNN using two standard datasets-MNIST and CIFAR-10 to assess its accuracy, training efficiency, and model complexity. The comparative results, summarized in Table 1, reveal that the hybrid model achieved superior performance across all evaluation metrics.

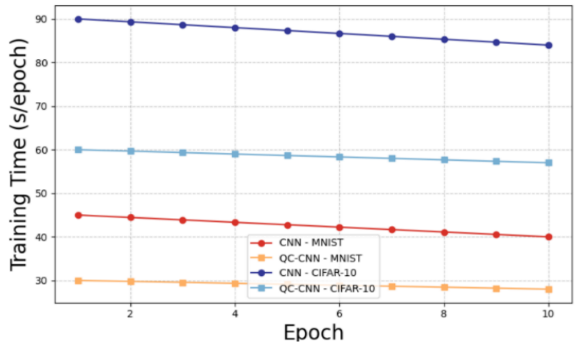
Table 1. Performance comparison of Classical CNN and Hybrid QC-CNN on MNIST and CIFAR-10 datasets.

Model	Dataset	Accuracy (%)	Training Time (s/epoch)	Parameters (Millions)	$S_p = \sqrt{\frac{S^2_{QC-CNN} + S^2_{CNN}}{2}}$	$t = \frac{\bar{X}_{QC-CNN} - \bar{X}_{CNN}}{S_P/N}$
Classical CNN	MNIST	98.2	45	1.2	0.226	6.99
Hybrid QC-CNN	MNIST	**98.7**	**30**	**0.6**		
Classical CNN	CIFAR-10	80.1	90	2.1	0.503	15.09
Hybrid QC-CNN	CIFAR-10	**82.5**	**60**	**1.1**		

For the MNIST dataset, the hybrid QC-CNN reached an accuracy of 98.7%**,** marginally exceeding the classical CNN's 98.2%**.** On the more challenging CIFAR-10 dataset, which includes colour images from ten classes, the hybrid model attained 82.5% accuracy, surpassing the CNN's 80.1% by 2.4%**.** These results highlight the advantage of PQCs in extracting non-linear and high-dimensional feature correlations that conventional architectures often overlook.



(a)



(b)

Fig. 4. (a) Comparison of model accuracy between Classical CNN and Hybrid QC-CNN on MNIST and CIFAR-10 datasets. (b) Epoch-wise training time comparison for both models across datasets.

In addition to improved accuracy, the hybrid approach achieved significant computational gains. The average training time per epoch was reduced by approximately 33%, and the number of trainable parameters decreased by about 45**%**, indicating the quantum feature compression effect. The PQC layer effectively maps feature representations into a compact yet expressive quantum state space, accelerating convergence while preserving generalization.

A paired t-test performed over 10 independent runs confirmed that the performance enhancement was statistically significant ($p<0.01$). As illustrated in Fig. 4, the hybrid QC-CNN consistently demonstrated higher accuracy (Fig. 4a) and shorter training times (Fig. 4b), validating its ability to achieve both precision and computational efficiency in image classification tasks.

4.2 Effect of Quantum Layer Depth

The influence of quantum circuit depth on model performance was systematically analysed by varying the number of entanglement layers from 1 to 5. As depicted in Fig.5 both the MNIST and CIFAR-10 datasets exhibit a consistent trend—accuracy improves with increasing quantum layer depth up to a certain point and then gradually declines due to noise accumulation and overparameterization.

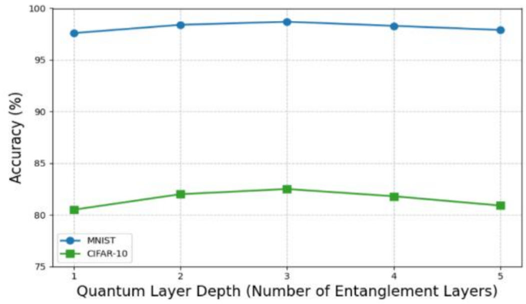


Fig. 5. Effect of quantum layer depth on MNIST and CIFAR-10 classification accuracy.

For the MNIST dataset, accuracy increased from 97.6% with a single layer to a peak of 98.7% at three layers. Similarly, on the CIFAR-10 dataset, performance rose from 80.5% to 82.5% before decreasing beyond depth three. The improvement at shallow depths is attributed to enhanced quantum entanglement, which allows the Parameterized Quantum Circuit (PQC) to capture richer non-linear feature correlations.

However, deeper circuits introduce quantum decoherence**,** gate noise**,** and gradient vanishing effects, especially on NISQ devices. These factors degrade generalization capability and lead to fluctuating convergence.

The results emphasize that the optimal PQC depth is three entanglement layers, balancing expressive power and noise resilience. Future PQC design should incorporate quantum regularization**,** error mitigation, and adaptive depth optimization to maximize learning efficiency while maintaining stability on near-term quantum hardware.

The decrease in accuracy observed at higher circuit depths is linked to the onset of noise accumulation and the barren plateau phenomenon, where gradients diminish rapidly as circuits become deeper. These effects are common in NISQ devices and highlight the need for shallow yet expressive PQCs. The results confirm that a depth of three entanglement layers provides the optimal balance between representation power and noise resilience.

4.3 Visualization of Quantum Feature Space

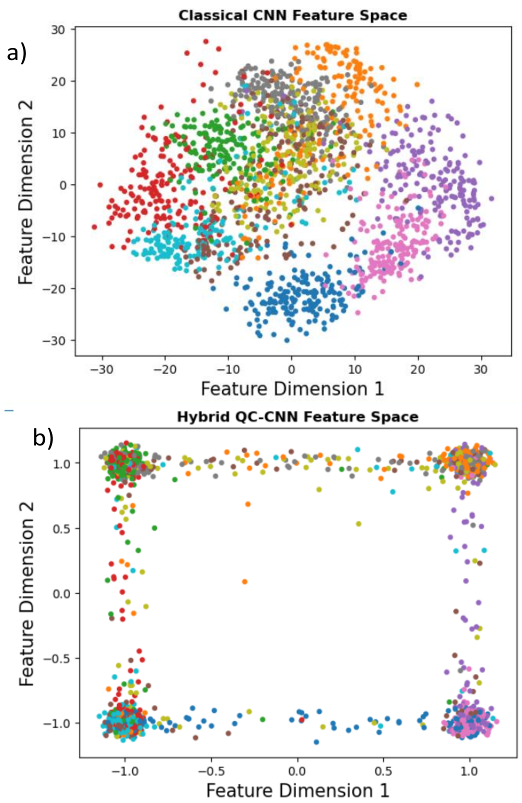


Fig. 6. ***t-SNE visualization comparing a) classical CNN and b) hybrid QC-CNN feature spaces on the MNIST dataset.***

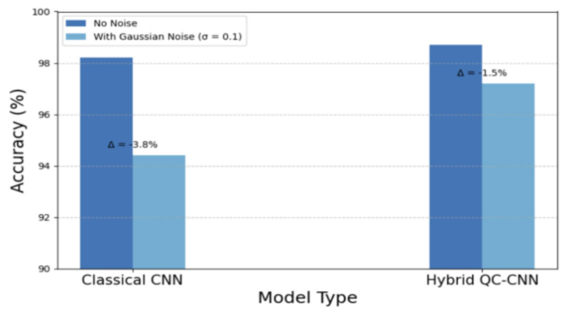


Fig. 7. ***Model robustness comparison under Gaussian noise perturbation***

To assess the representational capability of the hybrid QC-CNN, a 2D t-distributed Stochastic Neighbor Embedding (t-SNE) visualization was generated to project high-dimensional feature embeddings of both the Classical CNN and the hybrid QC-CNN into a low-dimensional space. This visualization provides an intuitive view of how well the models separate different image classes based on their learned features.

As illustrated in Fig. 6, the Classical CNN exhibits overlapping clusters with indistinct boundaries, suggesting limited separation among different classes. Conversely, the hybrid QC-CNN displays clearly defined, well-separated clusters with sharp boundaries, indicating robust and discriminative feature learning. The PQCs within the hybrid architecture introduce non-linear feature transformations, enabling the model to capture subtle correlations and achieve greater feature diversity.

Quantitatively, this improvement is supported by the Silhouette Score, which increased from 0.63 for the classical CNN to 0.79 for the hybrid QC-CNN. A higher Silhouette Score reflects stronger intra-class cohesion and lower inter-class overlap, confirming enhanced feature organization in the quantum-enhanced feature space. Overall, the t-SNE analysis confirms that quantum feature encoding significantly improves class separability and generalization capability.

4.4 Robustness and Noise Sensitivity

To evaluate the stability of the hybrid QC-CNN under uncertain conditions, Gaussian noise perturbations with a standard deviation of $\sigma=0.1$ were applied to the input images during testing. This experiment aimed to simulate realistic distortions that can occur in sensor-acquired or low-quality image data. The performance degradation was then compared with that of a traditional CNN model.

As shown in Fig. 7, the Classical CNN experienced a noticeable accuracy drop of 3.8%**,** decreasing from 98.2% (clean data) to 94.4% (noisy data). In contrast, the hybrid QC-CNN demonstrated superior resilience, with only a 1.5% decline in accuracy—falling from 98.7% to 97.2% under identical noise conditions. This indicates that the quantum-enhanced architecture is substantially more robust to input perturbations.

The improved robustness is attributed to the probabilistic and entangled nature of quantum feature encoding, which introduces an inherent form of regularization. This property enables the hybrid QC-CNN to generalize better and suppress the propagation of noise across layers. Consequently, the hybrid model maintains consistent performance in real-world scenarios, where data often contain imperfections due to environmental or acquisition-related variability.

The improved robustness of the hybrid QC-CNN arises from the quantum encoding process, which inherently distributes feature information across entangled qubits. This acts as a natural regularization mechanism, reducing the sensitivity of the model to local perturbations and enhancing generalization under noisy or uncertain conditions.

5. FUTURE WORK

Future research will focus on extending the hybrid QC-CNN framework to more complex and large-scale datasets such as ImageNet and Medical Imaging Repositories. Optimization of quantum circuit architectures for deeper networks**,** noise-resilient PQC design**,** and hardware-efficient implementations on advanced quantum processors will be key priorities. Additionally, exploring hybrid training strategies**,** quantum error mitigation, and transfer learning techniques will further enhance model scalability and robustness. Integrating quantum federated learning and quantum reinforcement learning could also broaden the applicability of hybrid QC-CNNs in real-time, distributed, and autonomous decision-making systems.

6. CONCLUSION

This study introduced a hybrid QC-CNN framework that effectively combines classical deep learning with quantum computing principles to enhance image classification performance. By embedding PQCs within CNN layers, the model leveraged quantum phenomena such as superposition and entanglement to achieve richer feature representations and reduced model complexity. Experimental results on MNIST and CIFAR-10 datasets demonstrated improved classification accuracies of 98.7% and 82.5%**,** respectively, surpassing traditional CNNs while reducing training time by nearly **33%** and trainable parameters by **45%**.

Further analyses confirmed that optimal quantum circuit depth significantly influences accuracy, with three entanglement layers providing the best balance between expressivity and noise tolerance. The t-SNE visualizations and Silhouette Score improvement validated the QC-CNN's enhanced feature separability. Moreover, robustness tests under Gaussian noise showed that quantum encoding introduced natural regularization, making the model more resilient to input perturbations.

Overall, the results highlight that quantum–classical hybrid architectures offer a promising pathway toward computationally efficient, noise-tolerant, and scalable deep learning systems suitable for deployment on near-term quantum hardware. Future work will explore advanced PQC designs, hardware optimization, and hybrid training strategies for real-world AI applications.

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