The Trapezoidal Fuzzy Number for Solving Fuzzy Linear Programming Problem using Alpha-Cut Method

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ABSTRACT

Linear Programming (LP) problem is one of optimization problem. Based on its limited resources and other restriction, we find the optimal solution for the problem. LP problems have very wide applications in our daily problems. They deal with situations where a number of resources, such as men materials, machines, and land are available, and are to be combined to yield one or more products. LP deals with that class of programming problems for which all relations among the variables are linear. The relation must be linear both in the constraints and in the function to be optimized. There are some types of fuzzy LP problems. One type is the right sides of the constraints are fuzzy numbers. The other type is the coefficients of the objective function are the fuzzy numbers. The most complicated type is the right side, the coefficients of the variables and the coefficients of the objective function are fuzzy numbers. There are many types of fuzzy numbers. Two of them are trapezoidal fuzzy number and triangular fuzzy number (TFN). They are easy to be counted and to be implemented. In this paper trapezoidal fuzzy number (TrNF) is defined, where the method of subtraction and division has been modified. These modified method is exactly inverse of the addition and multiplication operators. There are some techniques to solve the fuzzy LP problems. To slove this problem , we use trapezoidal fuzzy numbers. Here we propose a new operation on trapezodal fuzzy number to solve fuzzy LP problem using interval arithmetic based on Alpha-cut. We construct an assumptions to slove the trapezoidal fuzzy number linear programming problem.

Keywords

Arithmetic interval,function Principle, fuzzy linear programming, trapezoidal fuzzy number.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in 1965 [14]. Zimmermann [16] first proposed the concept of fuzzy linear programming in 1978. Then Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. Tanaka et. al [12] adopted this concept for solving mathematical programming Problem. Compos and Vardegay [2] considered linear programming problems with fuzzy constraints and fuzzy co-efficient in both left and right hand of the constraints set. Maleki [7] introduced a new method for solving linear programming problem with vagueness in constraints by using ranking function. Pandian and Jayalakshmi [11] proposed a new method for solving fully fuzzy linear programming problem with fuzzy variables.

Jayalakshmi and Pandian [5] introduced a new method for finding an optimal fuzzy solution for fuzzy linear programming problems. Dwyer[4] first introduced interval arithmetic in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by Moore [8,9].

Fuzzy number was defined in various operations. These operations do not explicitly make it clear. The proposed definitions are intrinsically natural. As an abstract mathematical system, fuzzy mathematics is very little studied subject. The same comment is true for interval arithmetic which may be considered as degeneracy fuzzy arithmetic. The usual arithmetic operation on real numbers can be extended to the ones defined on fuzzy number by means of Zedeh`s extension Principle [14,15].According to Chen,S.H.[3] standard fuzzy arithmetic operation using function principle, we have $\tilde{A}$- $\tilde{A}$ ≠0 and $\frac{\tilde{A}}{\tilde{A}} \neq 1$ However, in optimization and many engineering applications, it can be desirable to have crisp values for $\tilde{A}$- $\tilde{A}$ and $\frac{\tilde{A}}{\tilde{A}}$ i.e.the crisp values 0 and1 respectively

In this paper we propose the standard fuzzy interval arithmetic operation to solve trapezoidal fuzzy number is modified only on subtraction and division .This paper aims to slove the fuzzy LP problem using Alpha-cut method. we construct an assumptions to solve the trapezoidal fuzzy number linear programming problem. In this paper section 2 deal with some basic definition of fuzzy set. section 3, New Modified method of Subtraction and Division on Trapezoidal fuzzy number and its properties are discussed. In section 4,Addition, Subtraction, Multiplication and division of trapezoidal fuzzy number α-cut method are discussed. In section 5, an application of this operation are discussed and numerical example is given. Concluding remarks are given in section 6.

2. BASIC CONCEPT

2.1 Definition [6]

Let A be a classical set μ_A (x) be a real valued function defined from R→ [0,1].A fuzzy setA^* with the function $\mu_A(x)$ is defined by $A^* = [(x, \mu_A(x)); x \in A$ and $\mu_A(x) \in [0,1]\}$.The function μ_A (x) is known as the membership function of A^*.

2.2 Definition [6]

Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$ the α-cut α_A is the crisp set α_A ={ x| A(x)≥ $\alpha\}$

2.3 Definition [14]

Given a fuzzy set A defined on X and any number$\alpha \in [0,1]$ the strong α- cut $\alpha_{+A}$is the crisp set α_{+A} ={ x| A(x)> $\alpha\}$

2.4 Definition

A fuzzy set A on R must possess at least the following three properties to verify as a fuzzy number,

(i) $\tilde{A}$ must be a normal fuzzy set;
(ii) $\alpha_{\tilde{A}}$ must be closed interval for every $\alpha \in [0,1]$
(iii) The support of $\tilde{A}$, $0_{\tilde{A}}$,must be bounded.

2.5 Definition: Trapezoidal Fuzzy number [13]

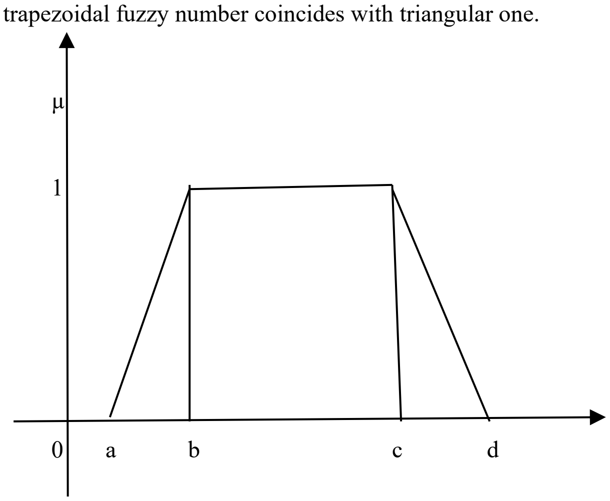
It is a fuzzy number represented with four vertices as follows: $\widetilde{A}$=(a, b, c, d) this representation is interpreted as membership functions and hold the following conditions

(i) a to b is strictly increasing function.
(ii) 1, where b=c
(iii) c to d is strictly decreasing function.
(iv) $a \leq b \leq c \leq d$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x<a \\ \frac{x-a}{b-a} & for\ a\leq x\leq b \\ 1 & for\ b\leq x\leq c \\ \frac{d-x}{d-c} & for\ c\leq x\leq d \\ 0 & for x>d \end{cases}$$

α- cut of a TrFN for this shape is denoted as A_α and is defined as

A_α = [(b –a)α +a , -(d-c)α +d] $\forall \alpha \in [0,1]$. When a=b ,the trapezoidal fuzzy number coincides with triangular one.



Trapezoidal Fuzzy Number $\widetilde{A}$=(a, b, c, d)

2.6 Definition

Positive Trapezidal Fuzzy Number:A positive trapezoidal fuzzy number is a fuzzy number $\tilde{A}$ is defined as

$\tilde{A}$ **= (a_1, a_2, a_3, a_4) where all $a_i > 0, \forall\, i = 1,2,3,4$.**

2.7 Definition

Negative Trapezidal Fuzzy Number:A negative trapezoidal fuzzy number is a fuzzy number $\tilde{A}$ is defined as

$\tilde{A}$ = (a_1, a_2, a_3, a_4) where all $a_i < 0, \forall\, i = 1,2,3,4$.

Remark: A negative trapezoidal fuzzy number can be written as the negative multiplication of a positive trapezoidal fuzzy number.

2.8 Definition

Equal Trapezoidal Fuzzy Number: Let $\tilde{A}$ = (a_1, a_2, a_3, a_4) and $\tilde{B}$ = (b_1, b_2, b_3, b_4) be two trapezoidal fuzzy number. If $\tilde{A}$ is identically equal to $\tilde{B}$ only if $a_1 = b_1, a_2 = b_2, a_3 = b_3$ and $a_4 = b_4$.

2.9 Operation of trapezoidal fuzzy number using Function Principle

The following are the four operations that can be performed on TrFN: Let $\tilde{A}$ = (a_1, a_2, a_3, a_4) and $\tilde{B}$ = (b_1, b_2, b_3, b_4) then,

(i) Addition: $\tilde{A}$ (+) $\tilde{B}$ = (a_1, a_2, a_3, a_4) (+)(b_1, b_2, b_3, b_4)

=($a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4$)

(ii) Subtraction : $\tilde{A}$ (-) $\tilde{B}$ = (a_1, a_2, a_3, a_4) (-)(b_1, b_2, b_3, b_4)

= ($a_1 - b_4, a_2 - b_2, a_3 - b_3, a_4 - b_1$)

(iii) Multiplication: $\tilde{A}$ (x) $\tilde{B}$ = (a_1, a_2, a_3, a_4) (x)(b_1, b_2, b_3, b_4)

=(min($a_1b_1, a_1b_4, a_4b_1, a_4b_4$), a_2b_2, a_3b_3, max ($a_1b_1, a_1b_4, a_4b_1, a_4b_4$))

(iv) Division : : $\widetilde{A}$ (÷)$\tilde{B}$ = (a_1, a_2, a_3, a_4) (÷)(b_1, b_2, b_3, b_4)

=($min(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}), \frac{a_2}{b_2}, \frac{a_3}{b_3}$,max ($\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}$))where $\forall b_i \neq 0$

2.10 For example

Let $\tilde{A}$ = (2,4,6,8) and $\tilde{B}$=(1,2,3,4) be two fuzzy numbers,then

(i)$\tilde{A} + \tilde{B}$ =(3,6,9,12) (ii) $\tilde{A} - \tilde{B}$ =(-2,2,3,7), (iii) $\tilde{A}$ (x) $\tilde{B}$ =(2,8,18,32) and (iv) $\widetilde{A}$ (÷)$\tilde{B}$ =(0.5,1.33,3,8)

(v) $\tilde{A}$- $\tilde{A}$ =(-6,0,0 ,6) and (vi) $\frac{\tilde{A}}{\tilde{A}}$ =(0.25,0.67,1.5,4)

Note

Form above its clear that $\tilde{A}$- $\tilde{A}$ ≠0 and $\frac{\tilde{A}}{\tilde{A}} \neq 1$,where o and 1 are singletons ,Fuzzy representation is (0,0,0,0) and (1,1,1,1).It is follows that the $\tilde{D}$ solution of the fuzzy linear equation have $\tilde{A} + \tilde{B} = \tilde{D}$ whose solution is not given by $\tilde{B} = \tilde{D} - \tilde{A}$

e.g $\tilde{A} + \tilde{B}$ =(3,6,9,12) = $\tilde{D}$

But (1,2,3,4)=(3,6,9,12) – (2,4,6,8) = (-5,2,3,10)≠ $\tilde{B}$.

The same condition appears when solving the fuzzy eqution $\tilde{A}$ x $\tilde{B}$ =$\tilde{D}$,whose solution is not given by $\tilde{B} = \frac{\tilde{D}}{\tilde{C}}$.

For example $\tilde{A}$ x $\tilde{B}$ =(2,8,18,32) =$\tilde{D}$

But $\tilde{B}$= $\frac{(2,8,18,32)}{(2,4,6,8)}$ = (0.25, 1.33,4.5,16) ≠ $\tilde{B}$

Therefore the addition , subtraction and multiplication and division of fuzzy numbers are not reciprocal operation .Its is not possible to slove inverse problems exactly using the standard fuzzy arithmetic operations.

To overcome this in function principle operation of trapezoidal fuzzy number a new operation is proposed that follows exact inversion.

3. NEW MODIFIED METHOD OF SUBTRACTION AND DIVISION ON TRAPEZOIDAL FUZZY NUMBER

Aime of the objective is to develop a new subtraction and division operators on trapezoidal fuzzy number, Which are inverse operations of the addition and multiplication operators.

3.1 Subtraction

Let $\tilde{A}$ = (a_1, a_2, a_3, a_4) and $\tilde{B}$ = (b_1, b_2, b_3, b_4) then, $\tilde{A}$ - $\tilde{B}$ =($a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4$).

The subtraction operation exist only if the following condition is satisfied $DP(\tilde{A}) \geq DP(\tilde{B})$where

DP $(\widetilde{A})$=$\frac{a_4-a_1}{2}$ and DP$(\widetilde{B})$=$\frac{b_4-b_1}{2}$,where DP denote difference point of trapezoidal fuzzy number.

3.2 Division

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ then, $\frac{\tilde{A}}{\tilde{B}}$ =$(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$, where $\forall b_i \neq 0$

The new division operation exists only if the following conditions are satisfied $\left[\frac{DP(\tilde{A})}{MP(\tilde{A})}\right] \geq \left[\frac{DP(\tilde{B})}{MP(\tilde{B})}\right]$ and

the negative trapezoidal fuzzy number should be changed into negative multiplication of positive number as by definition 2.7.

where MP$(\widetilde{A})$=$\frac{a_4+a_1}{2}$,DP $(\widetilde{A})$=$\frac{a_4-a_1}{2}$ and MP$(\widetilde{B})$=$\frac{b_4+b_1}{2}$,DP$(\widetilde{B})$=$\frac{b_4-b_1}{2}$ where MP denotes midpoint and DP denotes difference point of a trapezoidal fuzzy number.

3.3 Condition of Subtraction and Division Operator

(i) Inverse operation of addition : $\tilde{B} + (\tilde{A} - \widetilde{B}) = (\widetilde{A - B}) + \tilde{B}$

(ii) Multiplication by a scalar : p$(\tilde{A} - \widetilde{B})$=$(p\tilde{A} - p\tilde{B})$

(iii)Neutral element: $(\tilde{A} - 0) = \tilde{A}$

(iv) Associatively: $(\widetilde{A - B}) + \tilde{\tilde{C}} = \tilde{A} - (\tilde{B} - \tilde{C})$

(v)Inverse element: Any fuzzy number is own inverse under the modified subtraction.i.e $\tilde{A} - \tilde{A} = 0$.

(vi) Regularity: $\widetilde{A - B}$ =$\tilde{A} - \tilde{C}$ implies $\tilde{B}$ =$\tilde{C}$

(vii) Psedu –distributivity with addition:$(\tilde{A} + \tilde{B})$ - $(\tilde{C} + \tilde{D})$ =$(\tilde{A}$ - $\tilde{C})$ + $(\tilde{B}$ -$\tilde{D})$

(vii)Inverse operation of multiplication: $\tilde{B} \times \frac{\tilde{A}}{\tilde{B}}$=$\frac{\tilde{A}}{\tilde{B}} \times \tilde{B}$ =$\tilde{A}$

(ix)Neutral element: The singleton $\tilde{1}$ =(1,1,1,1) defined by a constant profile equal to $\tilde{1}$ is

$\frac{\tilde{A}}{\tilde{1}}$=$(a_1, a_2, a_3, a_4)$ =$\tilde{A}$

(x)Inverse element : For any fuzzy number is its own inverse under the modified division operator

$\frac{\tilde{A}}{\tilde{A}}$ = (1,1,1,1)

(xi) Regularity: $\frac{\tilde{A}}{\tilde{C}}$=$\frac{\tilde{A}}{\tilde{B}}$ implies $\tilde{B} = \tilde{C}$

(xii) Distributivity with regard to addition : $\frac{(\widetilde{A+B})}{\tilde{C}} = \frac{\tilde{A}}{\tilde{C}} + \frac{\tilde{B}}{\tilde{C}}$

3.4 Necessary Existence condition for Subtraction and Division

Proposition 3.1

The subtraction operation exist only if the following condition is satisfied $DP(\tilde{A}) \geq DP(\tilde{B})$where

DP $(\widetilde{A})$=$\frac{a_4-a_1}{2}$ and DP$(\widetilde{B})$=$\frac{b_4-b_1}{2}$,where DP denote difference point of trapezoidal fuzzy number

Proof: From the definition 2.5 (i) ,(ii) and (iii) are gives when (iv) satisfies.Now we have to derive the necessary existence for $\tilde{A} - \tilde{B}$ which is equal to (c_1, c_2, c_3, c_4)

Let $c_1 \leq c_2 \leq c_3 \leq c_4 \Rightarrow c_1 \leq c_4 \Rightarrow a_1 - b_1 \leq a_4 - b_4$

$\Rightarrow b_4 - b_1 \leq a_4 - a_1$

$\Rightarrow$(MP$(\widetilde{B})$ + DP$(\widetilde{B})$) –(MP$(\widetilde{B})$ - DP$(\widetilde{B})$)≤ (MP$(\widetilde{A})$ + DP$(\widetilde{A})$) –(MP$(\widetilde{A})$ -DP$(\widetilde{A})$)

$\Rightarrow$MP$(\widetilde{B})$ + DP$(\widetilde{B})$) –MP$(\widetilde{B})$ +DP$(\widetilde{B})$ ≤ MP$(\widetilde{A})$ + DP$(\widetilde{A})$) –MP$(\widetilde{A})$ +DP$(\widetilde{A})$

$\Rightarrow$2DP $(\widetilde{B})$)≤ 2DP $(\widetilde{A})$

$\Rightarrow$DP $(\widetilde{B})$)≤ DP $(\widetilde{A})$

Which is the necessary existence condition of new subtraction operator.

Proposition 3.2

The new division operation exists only if the following conditions are satisfied $\left[\frac{DP(\tilde{A})}{MP(\tilde{A})}\right] \geq \left[\frac{DP(\tilde{B})}{MP(\tilde{B})}\right]$

Proof:

From the definition 2.5 (i),(ii) and (iii) are gives when (iv) satisfies.Now we have to derive the necessary existence for $\tilde{A} - \tilde{B}$ which is equal to (c_1, c_2, c_3, c_4)

Let $\boldsymbol{c_1 \leq c_2 \leq c_3 \leq c_4 \Rightarrow c_1 \leq c_4}$

$\Rightarrow a_1/b_1 \leq a_4/b_4 \Rightarrow a_1 b_4 \leq a_4 b_1$

$\Rightarrow$ (MP$(\widetilde{A})$ - DP$(\widetilde{A})$) (MP$(\widetilde{B})$ +DP$(\widetilde{B})$)≤ (MP$(\widetilde{A})$ + DP$(\widetilde{A})$) (MP$(\widetilde{B})$ -DP$(\widetilde{B})$)

$\Rightarrow$MP$(\widetilde{A})$MP$(\widetilde{B})$+MP$(\tilde{A})$DP$(\widetilde{B})$-DP$(\tilde{A})$(MP$(\widetilde{B})$- DP$(\tilde{A})$DP$(\widetilde{B})$)≤MP$(\widetilde{A})$MP$(\widetilde{B})$-MP$(\tilde{A})$DP$(\widetilde{B})$ +DP$(\tilde{A})$(MP$(\widetilde{B})$ - DP$(\widetilde{A})$DP$(\widetilde{B})$

$\Rightarrow$ MP$(\tilde{A})$DP$(\widetilde{B})$ -DP$(\tilde{A})$ (MP$(\widetilde{B})$ ≤ DP$(\widetilde{A})$MP$(\widetilde{B})$- MP$(\tilde{A})$DP$(\widetilde{B})$

$\Rightarrow$ 2MP$(\tilde{A})$DP$(\widetilde{B})$ ≤ 2DP$(\tilde{A})$ (MP$(\widetilde{B})$

$\Rightarrow$ MP$(\tilde{A})$DP$(\widetilde{B})$ ≤ DP$(\tilde{A})$ (MP$(\widetilde{B})$

$\Rightarrow \frac{\widetilde{DP(B)}}{MP(\widetilde{B})} \leq \frac{\widetilde{DP(A)}}{MP(\widetilde{A})}$

In this $\tilde{B}$ may positive or negative. So we take the condition $\left[\frac{DP(\tilde{A})}{MP(\tilde{A})}\right] \geq \left[\frac{DP(\tilde{B})}{MP(\tilde{B})}\right]$

Which is the necessary condition for new division.

3.5 Numerical Examples

1. Let $\tilde{A}$ = (5,6,7,8)

MP$(\tilde{A})$ = $\frac{5+8}{2}$ = 6.5 ; DP$(\tilde{A})$ = $\frac{8-5}{2}$ = 1.5

(1)Subtraction :

Now DP$(\tilde{A})$ = 1.5 therefore DP$(\tilde{A})$ = DP$(\tilde{A})$ .Hence $\tilde{A}$-$\tilde{A}$ is satisfying the condition

$\widetilde{A}$- $\widetilde{A}$ =($a_1 - a_1, a_2 - a_2, a_3 - a_3, a_4 - a_4$)=(5-5.6-6,7-7,8-8) =(0,0,0,0)

(ii)Divide:

Since $\left[\frac{DP(\widetilde{A})}{MP(\widetilde{A})}\right] = \left[\frac{1.5}{6.5}\right]$ =0.231 therefore $\left[\frac{DP(\widetilde{A})}{MP(\widetilde{A})}\right] = \left[\frac{DP(\widetilde{A})}{MP(\widetilde{A})}\right]$ .Hence $\frac{\tilde{A}}{\tilde{A}}$ satisfying the condition. $\frac{\tilde{A}}{\tilde{A}}$ =(($\frac{5}{5}, \frac{6}{6}, \frac{7}{7}, \frac{8}{8}$) = (1,1,1,1)

2. $\tilde{B}$ = (-5,-3,-2,-1)

MP($\tilde{B}$) = $\frac{-1-5}{2}$ = -3.5 ; DP($\tilde{B}$) = $\frac{-1+5}{2}$ = 2

(i) Subtraction :

Now DP($\tilde{B}$) = 2 therefore DP($\tilde{B}$) = DP($\tilde{B}$) .Hence $\widetilde{B}$- $\widetilde{B}$ is satisfying the condition

$\widetilde{B}$- $\tilde{B}$=(-5+5,-3+3.-2+2,-1+1) =(0,0,0,0).

(ii)Divide:

Since $\left[\frac{DP(\widetilde{B})}{MP(\widetilde{B})}\right] = \left[\frac{2}{-3.5}\right]$ =0.571 therefore $\left[\frac{DP(\widetilde{B})}{MP(\widetilde{B})}\right] = \left[\frac{DP(\widetilde{B})}{MP(\widetilde{B})}\right]$ .Hence $\frac{\tilde{A}}{\tilde{A}}$ satisfying the condition. $\frac{\tilde{A}}{\tilde{A}}$ =($\frac{-5}{-5}, \frac{-3}{-3}, \frac{-2}{-2}, \frac{-1}{-1}$) = (1,1,1,1)

4. ARITHMETIC OPERATION OF FUZZY NUMBERS USING ALPHA – CUT METHOD [10]

In this section we consider Addition, Subtraction ,Multiplication and Division of fuzzy numbers using α-cut method.

4.1 Addition of Fuzzy Numbers

Let X=[a,b,c,d] and Y=[p,q,r,s] be two fuzzy numbers whose membership functions defined by

$$\mu_x(x) = \begin{cases} 0 & for\ x<a \\ \frac{x-a}{b-a} & for\ a\le x\le b \\ 1 & for\ b\le x\le c \\ \frac{d-x}{d-c} & for\ c\le x\le d \\ 0 & for x>d \end{cases}$$

$$\mu_y(x) = \begin{cases} 0 & for\ x<p \\ \frac{x-p}{q-p} & for\ p\le x\le q \\ 1 & for\ q\le x\le r \\ \frac{s-x}{s-r} & for\ r\le x\le s \\ 0 & for x>s \end{cases}$$

Then α_x = [(b-a)α +a ,d-(d-c)α] and α_y =[(q-p)α+p,s-(s-r)α] are the α-cuts of fuzzy numbers X and Y repectively. To calculate the addition of fuzzy numbers X and Y using arithmetic interval

$\alpha_x + \alpha_y =$ [(b-a)α +a ,d-(d-c)α] +[(q-p)α+p,s-(s-r)α]

= [a+p+(b-a +q-p)α,d+s-(d-c+s-r)α]

4.2 Subtraction of Fuzzy Numbers

Let X=[a,b,c,d] and Y=[p,q,r,s] are two fuzzy numbers.Then α_x = [(b-a)α +a ,d-(d- c)α] and α_y =[(q-p)α+p,s-(s-r)α] are the α-cuts of fuzzy numbers X and Y respectively.

To calculate subtraction of fuzzy numbers X and Y using arithmetic interval.

$\alpha_x - \alpha_y =$ [(b-a)α +a ,d-(d-c)α] -[(q-p)α+p,s- (s-r)α]

= [(a-p)+(b-a +s-r)α,(d-p)-(d-c+q-p)α]

4.3 Multiplication of Fuzzy Numbers

Let X=[a,b,c,d] and Y=[p,q,r,s] are two fuzzy numbers. Then α_x = [(b-a)α +a ,d -(d-c)α] and α_y =[(q-p)α+p,s-(s-r)α] are the α-cuts of fuzzy numbers X and Y respectively. To calculate the multiplication of fuzzy numbers X and Y using arithmetic interval.

$\alpha_x * \alpha_y =$ [(b-a) α +a, d-(d-c)α] * [(q-p) α+p, s-(s-r) α]

=[(b-a)α+a) *((q-p)α+p),(d-(d-c)α * (s-(s-r)α)]

4.4 Division of Fuzzy Numbers

Let X=[a,b,c,d] and Y=[p,q,r,s] are two fuzzy numbers. Then α_x = [(b-a)α +a ,d- (d-c)α] and α_y =[(q-p)α+p,s-(s-r)α] are the α-cuts of fuzzy numbers X and Y respectively. To calculate the division of fuzzy numbers X and Y using arithmetic interval.

$$\frac{\alpha_x}{\alpha_y} = \frac{[(b-a)\alpha+a\ ,d-(d-c)\alpha]}{[(q-p)\alpha+p,s-(s-r)\alpha]}$$

$$=\left[\frac{(b-a)\alpha+a}{s-(s-r)\alpha}, \frac{d-(d-c)\alpha}{(q-p)\alpha+p}\right]$$

4.5 Inverse of Fuzzy Numbers

To find inverse of the fuzzy numbers X. We first take the inveres of the α-cut of X is taken using arithmetic interval,

i.e $\frac{1}{\alpha_x} = \frac{1}{[(b-a)\alpha +a\ ,d-(d-c)\alpha]}$

$=\left[\frac{1}{d-(d-c)\alpha}, \frac{1}{(b-a)\alpha+a}\right]$

Assumption

To solve fuzzy liner programming problem the following assumption are used.

(1) Obtain the value of $\alpha_x$and α_y are the α-cut of fuzzy numbers X and Y respectively.

(2) Using arithmetic interval add $\alpha_x$and α_y.

(3) The values can found in (1) and (2)is transformed into a LP problem and formulated as:

Minimize(or Maximize) $c_1x_1 + c_2x_2 + \cdots, +c_nx_n$

Subject to $a_{11}x_1 + a_{12}x_2 + \cdots, +a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots, + a_{2n}x_n \le b_2$

… … … …

$a_{m1}x_1 + a_{m2}x_2 + \ \ldots, a_{mn}x_n \le b_n$

Simply we write

Max Z = $\sum_{j=1}^{n} c_j x_j$

Subject to,

$\sum_{j=1}^{n} a_{ij}x_j \le b_i$, i=1 ,2, …m

$x_j \ge 0$,j =1,2,…,n

The function Max Z is called an objective function .The matrix $[a_{ij}]$ is called a constraint matrixa, the vector $b_i = (b_1, b_2, \ldots b_m)$is called right hand vector and $c_j = (c_1, c_2, \ldots c_n)$ is called a cost vector.

(4) Solving the above LP Problem, We get the required optimal Solution.

5. NUMERICAL EXAMPLE

Here we are going to slove fully fuzzy LP problem using α – cut method.

Max Z = 6x + 5y

Subject to

[(1,2,3,4)+(3,4,5,6)]x+[(4,5,6,7)+(1,2,3,4)]y ≤[(12,14,16,18) +(16,18,20,22)]

[(1,2,3,4)+(3,4,5,6)]x +[(0,1,2,3)+(2,3,4,5]y ≤ [(14,16,18,20) +(18,20,22,24)]

x ,y ≥ 0

(1) Determine α –cut of $\alpha_x$and α_y.

The α –cut of the fuzzy trapezoidal number (1,2,3,4) is

$$\alpha_x = [\ \alpha +1 ,4\text{-}\alpha\ ]$$

The α –cut of fuzzy trapezoidal numbers (3,4,5,6) is

$$\alpha_y = [\alpha+3 , 6\text{-}\alpha]$$

(2) Using arithmetic interval add $\alpha_x$and α_y.

$$\alpha_x + \alpha_y = [\alpha + 1{,}4 - \alpha] + [\alpha + 3{,}6 - \alpha] = 14$$

In this way the constraint matrix a_{ij} and the right hand number b_i are

$$a_{11} = [(1,2,3,4) + (3,4,5,6)] = 14$$

$$a_{12} = [(4,5,6,7) + (1,2,3,4)] = 17$$

$$a_{21} = [(1,2,3,4) + (3,4,5,6)] = 14$$

$$a_{22} = [(0,1,2,3) + (2,3,4,5)] = 10$$

$$b_1 = [(12,14,16,18) + (16,18,20,22)] = 68$$

$$b_2 = [(14,16,18,20) + (18 ,20 ,22,24)] = 76$$

(3) Now the LP problem becomes as

Max Z= 6x +5y

Subject to,

$$14x + 17y \leq 68$$

$$14x + 10y \leq 76$$

$$x ,y \geq 0$$

(4)From the above LP problem we get the optimal solution

Max Z= 20, x=0 ,y=4.

6. CONCLUSION

The paper aims to introduce a new interval arithmetic operation of subtraction and division. The modified method is exactly inverse of the addition and multiplication operators .Based on the result above, it can be concluded that to solve the fuzzy LP problem, with trapezoidal fuzzy number is solved by using α – cut operation without transformed them into a classical LP problem. This is an quite easy approach compared to earlier approaches for solving fuzzy linear programming problem using interval arithmetic. These operations may help us to solve many optimization problems.

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