Optimal Statistical Classifier for UAV Detection: Application of Kaczmarz & Rank Two RLS Methods

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ABSTRACT

The growing prevalence of unmanned aerial vehicles (UAVs), also known as drones, has intensified the demand for robust detection technologies for security and counterterrorism applications. Acoustic sensors provide a promising solution for UAV detection, although their effectiveness is strongly influenced by environmental noise, making advanced signal processing techniques necessary. Most UAV propulsion emission is concentrated within a specific frequency band, making it possible to extract dominant frequencies for detection. The intensity of this band can be modeled as a stochastic process defined by the sum of normally distributed amplitudes across the selected frequencies. Detection performance depends on how well the UAV intensity distribution can be distinguished from the background noise distribution. The problem can therefore be formulated as a hypothesis test between noise-only and UAV-present conditions. Classification accuracy is improved by selecting appropriate frequency components and optimizing amplitude estimation methods to reduce overlap between the two distributions.
Algorithm selection is crucial for classifier performance. This paper uses the Kaczmarz projection method as the primary algorithm for amplitude estimation within the selected frequency cluster. The method avoids matrix inversion and associated singularity problems, supports closely spaced frequencies and has linear computational complexity. However, the lack of tunable parameters can produce noisy estimates.
The recursive least squares (RLS) algorithm with rank two updates offers adjustable window size and forgetting factor parameters but has quadratic computational complexity. Moreover, ill-conditioning of the information matrix limits parameter selection and reduces the separation between UAV and noise distributions compared with the Kaczmarz method. Despite this limitation, RLS achieves better noise cancellation because of its adaptive parameter control. The proposed classifier therefore combines RLS-based noise suppression with Kaczmarz-based dominant frequency extraction. The approach is validated using real acoustic measurements of UAVs and environmental background noise.

Keywords

Identification of dominant frequency clusters in UAV data, hypothesis test, optimal statistical classifier for UAV detection, Kaczmarz projection method, least mean squares algorithm, rank two RLS method, noise cancellation

1. OVERVIEW OF THE DETECTION METHOD

The development of detection systems for UAVs (unmanned aerial vehicles), known as drones, is critically important for preventing terrorist attacks, as it enhances situational awareness and supports timely countermeasures, [1]. Acoustic sensors use flight noise analysis to detect drones, offering a simple, cost-effective complement to other counter-UAV technologies, [2] - [18]. Because acoustic sensors operate within a limited range and are prone to background noise, there is a clear need for advanced signal processing techniques to improve detection performance in both standalone and hybrid systems.
Statistical approaches are the most promising for UAV detection due to the stochastic and noisy nature of acoustic signals. In contrast to deterministic techniques, probabilistic modeling and analysis provide a broader set of tools for building robust detection systems, [19].
Quadcopter acoustic emissions comprise closely spaced tonal components, harmonics, beat-frequency modulation and broadband aerodynamic noise, resulting in a spectrum of narrow frequency bands/clusters instead of a single discrete tone. Therefore, identifying dominant frequencies within these bands is central to UAV detection. In dominant UAV frequency extraction, the band intensity is modeled as a stochastic process resulting from the superposition of normally distributed amplitudes over the band frequencies, see section 2.1. Detection performance is determined by the statistical divergence between two normal distributions representing the UAV intensity signal and the intensity of the background noise within the band, see Figure 2(a). In other words, the detection problem is cast as a hypothesis testing framework, [19],[20] where the null hypothesis assumes that the sample is drawn from the normal distribution of background noise intensity, and the alternative hypothesis assumes that it is drawn from the UAV intensity distribution, see section 3.
The optimal classifier is determined by the selection of frequencies within the band and the design and parameterization of amplitude estimation algorithms, such that the overlap between the noise and UAV intensity distributions is minimized, see section 3.1.
The choice of the algorithms plays important role in the classifier. The Kaczmarz projection method,[20] -[23] is used in this paper as the primary algorithm for estimating amplitudes within the frequency band, characterized by the absence of matrix inversion (eliminating singularity issues), flexibility in selecting closely spaced frequencies and linear computational complexity. In the classical Kaczmarz algorithm, the model output is updated to match the measured signal at each iteration, see section 4.1, but the absence of

tunable parameters may lead to noisy estimates.
The second algorithm is the RLS method with rank two updates, which exhibits quadratic complexity and enables tuning of the window size and forgetting factor to control estimation performance, see section 4.2, [24] -[26]. However, parameter selection is limited by the ill-conditioning of the information matrix. Consequently, it achieves weaker separation between noise and UAV intensity distributions in the dominant frequency extraction approach than the Kaczmarz method, see Figure 1. However, the RLS algorithm achieves better noise cancellation performance (as noted in many studies), owing to its flexible window size and forgetting factor, compared to the Kaczmarz method.
The highest classifier accuracy is achieved using the multi-band estimation scheme [26], where low frequency background noise is mitigated using RLS method and higher-frequency UAV signatures are extracted via the Kaczmarz algorithm, see section 4.3 and Figure 3(a) and 3(b). However, noise cancellation methods are less robust than the UAV dominant frequency extraction method under variable background noise conditions.
Conventional methods, such as Kalman filtering, can also be used for UAV signature detection, [10]. However, the proposed approach relies on optimal classification rather than optimal filtering. By jointly analyzing amplitudes across relevant frequencies, it produces a stronger intensity-based detection signal and significantly outperforms Kalman filtering. The method is also easier to calibrate and does not require stochastic modeling or prior knowledge of process and measurement noise.
Acoustic drone detection can also be performed using machine learning techniques such as Support Vector Machines (SVMs), Convolutional Neural Networks (CNNs) and other deep learning models, [2],[3],[5], [7] -[18]. CNNs can automatically extract features from spectrograms and distinguish drone sounds from background noise. Although these methods can improve detection accuracy and adaptability, they require large training datasets, are sensitive to noise, may overfit to specific environments and demand considerable computational resources, limiting their suitability for battery-powered real-time embedded systems.
The approach presented in this paper returns drone detection to proven classical methods that offer greater interpretability and theoretical transparency than many modern machine learning techniques. It introduces an explainable framework for deriving an optimal statistical classifier and provides a theoretical justification of the best achievable performance within the selected algorithmic structure. The framework is based on adaptive signal processing, system identification and established statistical methodologies, including hypothesis testing, which reduces data requirements and improves suitability for computationally constrained, battery-powered real-time systems. The approach is tested using real world UAV acoustic data and environmental background noise recordings.

2. OPTIMAL CLASSIFIER

2.1 Band Intensity

The acoustic signal is partitioned into distinct frequency bands/clusters to enable evaluation of band-specific intensities. For each band, the intensity $I_{\text{band},k}$ is given by the sum of instantaneous normally distributed amplitudes $A_{j,k}$:

$$\boxed{I_{\text{band},k} = \sum_{j \in \text{band}} A_{j,k}} \tag{1}$$

in each step $k = 1, 2, \ldots$.
A frequency band may correspond either to UAV acoustic signatures or to background noise. For detection, bands are chosen according to the most pronounced UAV frequency components, such that $I_{\text{band},k}$ increases under UAV presence. Nevertheless, the choice of frequency bands must consider the statistical characteristics of the intensity and the spectral overlap between UAV signals and background noise.

2.2 Optimal Detection Threshold

The UAV signature intensity and background noise intensity are modeled as Gaussian random variables with means μ_s and μ_n, and variances σ_s^2 and σ_n^2, respectively. Consider the intersection of their corresponding probability density functions as follows, [19] :

$$\boxed{\frac{1}{\sigma_n} \exp\left[-\frac{(T-\mu_n)^2}{2\sigma_n^2}\right] = \frac{1}{\sigma_s} \exp\left[-\frac{(T-\mu_s)^2}{2\sigma_s^2}\right]} \tag{2}$$

where the detection threshold T is introduced for minimization of the total error. Optimal detection threshold can be calculated as the solution of the following equation:

$$\frac{1}{\sigma_s} \exp\left(-\frac{(x-\mu_s)^2}{2\sigma_s^2}\right) = \frac{1}{\sigma_n} \exp\left(-\frac{(x-\mu_n)^2}{2\sigma_n^2}\right)$$

with respect to unknown variable x. The equation can be solved by taking logarithms

$$-\frac{(x-\mu_s)^2}{2\sigma_s^2} - \ln\sigma_s = -\frac{(x-\mu_n)^2}{2\sigma_n^2} - \ln\sigma_n$$

and rearranging terms, which gives a quadratic equation with respect to x:

$$\frac{(x-\mu_n)^2}{\sigma_n^2} - \frac{(x-\mu_s)^2}{\sigma_s^2} = 2\ln\frac{\sigma_s}{\sigma_n}$$

$$x^2\left(\frac{1}{\sigma_n^2} - \frac{1}{\sigma_s^2}\right) - 2x\left(\frac{\mu_n}{\sigma_n^2} - \frac{\mu_s}{\sigma_s^2}\right) + \left(\frac{\mu_n^2}{\sigma_n^2} - \frac{\mu_s^2}{\sigma_s^2}\right) = 2\ln\frac{\sigma_s}{\sigma_n} \tag{3}$$

The optimal threshold T, computed according to (3), is located between μ_n and μ_s and is illustrated in Figure 2(a) by the black line marking the intersection of the two distributions.

3. UAV DETECTION FRAMEWORK

The proposed UAV detection framework is formulated as the following statistical hypothesis testing problem :

$$\boxed{\begin{aligned} H_0 \text{ (noise only)} : I_{\text{band},k} &\sim \mathcal{N}(\mu_n, \sigma_n^2) \\ H_A \text{ (UAV present)} : I_{\text{band},k} &\sim \mathcal{N}(\mu_s, \sigma_s^2) \end{aligned}}$$

where H_0 denotes the null hypothesis that the intensity $I_{\text{band},k}$ follows the normal distribution associated with measurement noise and H_A denotes the alternative hypothesis that the sample originates from the UAV sound distribution.
With the optimal threshold T established in section 2.2, the corresponding type I (false alarm) and type II (miss) probabilities (α & β

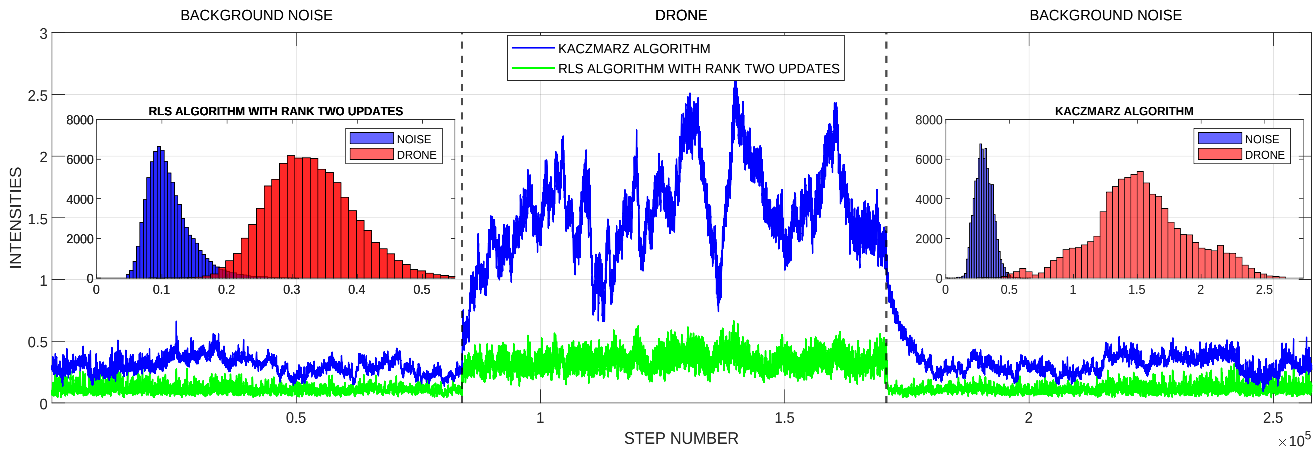
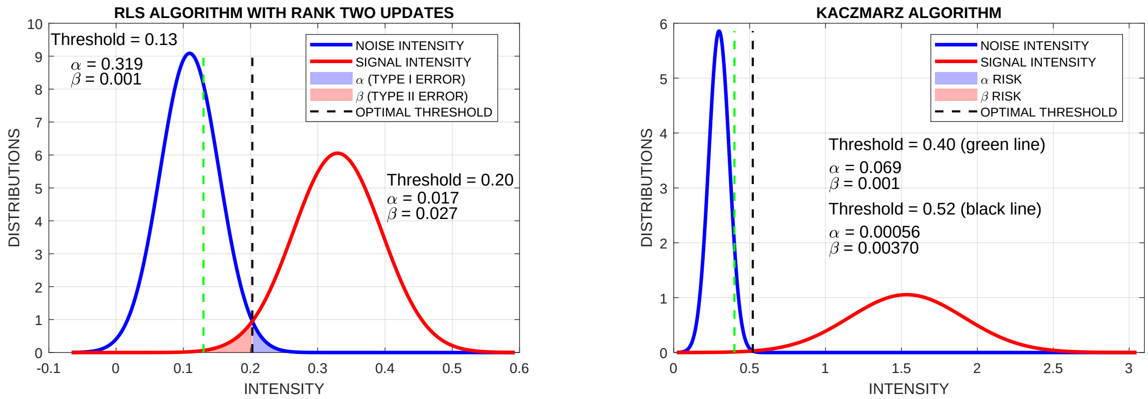


Fig. 1. The Figure presents UAV detection based on intensity estimation using Kaczmarz and RLS algorithms. The Figure shows a noise–drone–noise sequence with intensity estimates plotted in blue (Kaczmarz) and green (RLS).



(a) (b)

Fig. 2. Detection performance is assessed in Figures 2(a) and 2(b) using the $\alpha + \beta$ risks evaluated at the optimal threshold. The parameters of normal distributions are associated with histograms in Figure 1. The results indicate that the Kaczmarz algorithm produces significantly less overlap between the noise and UAV intensity histograms than the RLS method. Furthermore, for a fixed miss probability of $\beta = 0.001$ (green lines), the RLS method results in a considerably higher α risk.

risks) are obtained as follows, see Figure 2(a):

False alarm probability (type I error) :

$$\alpha = P(I_{\text{band},k} > T \mid H_0) = 1 - \Phi\left(\frac{T-\mu_n}{\sigma_n}\right) \tag{4}$$

Miss probability (type II error) :

$$\beta = P(I_{\text{band},k} \leq T \mid H_A) = \Phi\left(\frac{T-\mu_s}{\sigma_s}\right) \tag{5}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

3.1 Determination of Optimal Set of Frequencies via Minimization of $\alpha + \beta$

The optimal choice of the frequencies $q_i, i = 1, 2, ..., n$ in the band related to the UAV sounds is associated with minimization of the sum of the probability errors (α & β risks) for calculated optimal threshold T:

$$\begin{aligned} &(q_1, \cdots, q_n) \in \underset{q \in \text{band}}{\arg\min} (\alpha + \beta) \\ &\alpha + \beta = 1 - \Phi\left(\frac{T-\mu_n}{\sigma_n}\right) + \Phi\left(\frac{T-\mu_s}{\sigma_s}\right) \end{aligned} \tag{6}$$

In other words, the frequency is considered part of the band if the total probability error decreases, and this decrease is sufficiently significant. The mechanism can also be associated with maximization

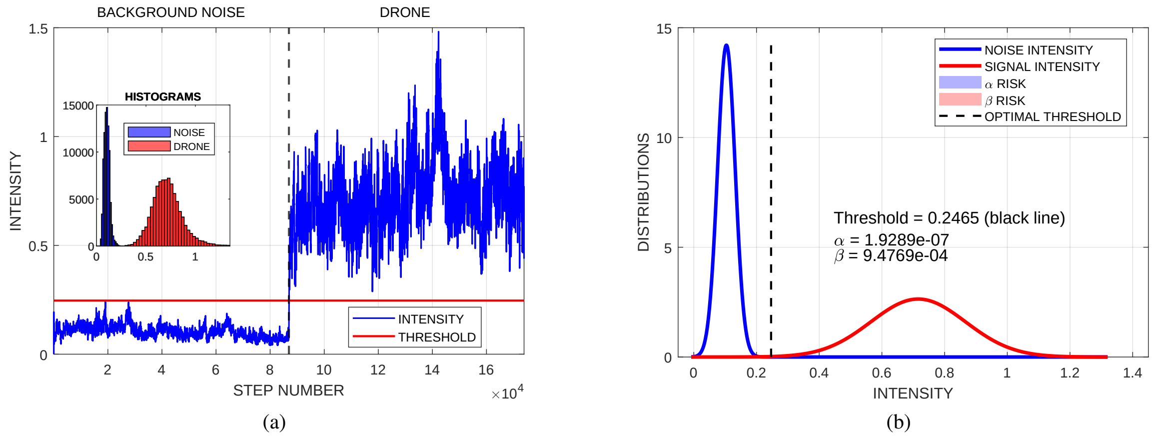


Fig. 3. Figure 3(a) illustrates UAV detection based on intensity estimation in a noise–drone sequence using the Kaczmarz algorithm together with an RLS-based noise cancellation scheme. Figure 3(b) shows the corresponding detection performance, where the optimal threshold (red line in Figure 3(a)) yields minimal overlap. The parameters of the distributions shown in Figure 3(b) correspond to the histograms presented in Figure 3(a).

of the overlap based power $P_{\text{overlap}} = -\log(\alpha + \beta)$.
Finally, Bhattacharyya distance, [27] which measures the divergence between two normally distributed populations can also be used for frequency selection instead of the overlap. Maximizing the Bhattacharyya distance approximately corresponds to minimizing $\alpha + \beta$.
Figures 1 and 2 illustrate UAV detection based on estimated intensity using two different algorithms: Kaczmarz and RLS, see section 4. In Figure 1, the signal is structured into three parts: noise, drone presence and noise again. The corresponding intensity estimates are shown as blue (Kaczmarz) and green (RLS) curves. Detection performance is evaluated using the total risk, defined as the sum of α and β, computed at the optimal decision threshold (see Figures 2(a) and 2(b)). The results show that the Kaczmarz algorithm provides better separation between noise and drone segments. In contrast, the RLS method produces a higher false-alarm risk α when the miss probability is fixed at $\beta = 0.001$.

4. RECURSIVE ALGORITHMS FOR ESTIMATION OF THE AMPLITUDES

Assume that the measured signal y_k and its corresponding model $\hat{y}_k$ are represented as follows:

$$y_k = \varphi_k^T \theta_* + \xi_k \tag{7}$$

$$\hat{y}_k = \varphi_k^T \theta_k \tag{8}$$

$$\varphi_k^T = [cos(q_1 k)\ sin(q_1 k)\ \cdots\ cos(q_h k)\ sin(q_h k)]$$

where θ_* is the vector of unknown parameters, φ_k is the harmonic regressor, $q_1, ..., q_h$ are the frequencies, and ξ_k is white Gaussian zero mean noise uncorrelated with φ_k, $k = 1, 2, ...$
Estimation accuracy and UAV detection performance depend on the adopted estimation algorithm, i.e., the vector of adjustable parameters θ_k. For amplitude estimation in (1), where the amplitudes are computed as the square root of the sum of squared coefficients, two algorithms were considered: the Kaczmarz projection method and the rank two RLS algorithm, introduced in sections 4.1 and 4.2, respectively.

4.1 Kaczmarz Projection Method

Equation (10) shows that, in the Kaczmarz algorithm (9), the model output $\varphi_k^T\ \theta_k$ exactly matches the measured signal y_k in each discrete step k:

$$\theta_k = \theta_{k-1} - \frac{\varphi_k}{\varphi_k^T\ \varphi_k}\,[\varphi_k^T\ \theta_{k-1} - y_k] \tag{9}$$

$$y_k = \varphi_k^T\ \theta_k \tag{10}$$

$$\varphi_k^T\ \tilde{\theta}_k = 0 \tag{11}$$

$$\varphi_k^T\ \varphi_k = h \tag{12}$$

A notable feature of the algorithm is the orthogonality property (11), whereby the parameter estimation error $\tilde{\theta}_k = \theta_k - \theta_*$ is orthogonal to the regressor vector. The harmonic regressor, formed by trigonometric functions of different frequencies, has a constant squared norm $\varphi_k^T\ \varphi_k$ that depends only on the number of frequencies. These properties have contributed to the successful application of Kaczmarz algorithms in automotive estimation problems, [20], [22].

Relation to LMS algorithm & convergence analysis. Least Mean Squares (LMS) algorithm, [28] can be written in the following form $\theta_k = \theta_{k-1} - \gamma\ \varphi_k\ [\varphi_k^T\ \theta_{k-1} - y_k]$. The choice of the step size $\gamma = (\varphi_k^T\ \varphi_k)^{-1} = 1/h$ in LMS algorithm which maximizes the error reduction per iteration according to the error model [22], $\|\tilde{\theta}_k\|^2 = \|\tilde{\theta}_{k-1}\|^2 - (2\gamma - \gamma^2 \varphi_k^T\ \varphi_k)(\varphi_k^T \tilde{\theta}_{k-1})^2$ relates Kaczmarz algorithm and normalized LMS algorithm. Inequality $\|\tilde{\theta}_k\|^2 \leq \rho^{\lfloor k/w \rfloor} \|\tilde{\theta}_0\|^2$, where $0 < \rho < 1$ and the floor function $\lfloor \cdot \rfloor$ rounds down to the nearest integer, which is obtained via summation of the errors over the window of the size w, gives explicit transient bound on estimation error for persistently exciting regressor and $\xi_k = 0$.
In this work, the Kaczmarz projection method is used as the main tool for amplitude estimation within the frequency band. It avoids matrix inversion, thereby circumventing singularity problems, allows flexible selection of closely spaced frequencies, and exhibits

linear computational complexity. A drawback, however, is the absence of tunable parameters, which can lead to noisy estimates.

4.2 RLS Algorithms with Rank Two Updates

Compared with conventional RLS algorithms, [21], [29],[30] rank two RLS offers a substantially more powerful framework for adaptive estimation in rapidly changing environments, [24] -[26]. By combining exponential forgetting with a sliding window architecture, the method achieves both high responsiveness and strong numerical stability. The sliding window mechanism simultaneously incorporates new measurements and removes outdated data, maintaining a fixed memory horizon. This enables rapid parameter convergence and highly agile tracking in non-stationary systems, while also mitigating singularities associated with obsolete measurements.
With a fixed history window of size w, rank two RLS avoids long-term round-off error buildup and yields more stable estimates. Importantly, the effects of outliers and poor initialization are guaranteed to vanish completely after a finite number of updates.
The forgetting factor $0 < \lambda < 1$ provides exponential weighting of historical data, while a composite piecewise forgetting profile, consisting of fast and slow decay regions offers fine-grained control over tracking speed, robustness and matrix conditioning, [25]. This framework also enables the incorporation of prior knowledge about system time scales and periodicities. These features suggest that rank two and low rank RLS methods are highly promising for UAV detection applications.
The rank two RLS algorithms can be written in the following form, [24] -[26]:

$$\Gamma_k = \frac{1}{\lambda} \left[\Gamma_{k-1} - \Gamma_{k-1} Q_k S_k^{-1} Q_k^T \Gamma_{k-1} \right] \tag{13}$$

$$\theta_k = \theta_{k-1} - \Gamma_{k-1} Q_k S_k^{-1} [Q_k^T \theta_{k-1} - \tilde{y}_k] \tag{14}$$

$$S_k = \lambda D + Q_k^T \Gamma_{k-1} Q_k \tag{15}$$

$$\tilde{y}_k^T = [y_k \sqrt{\lambda^w} y_{k-w}]$$

$$Q_k = [\varphi_k \sqrt{\lambda^w} \varphi_{k-w}]$$

where Γ_k and θ_k are estimates of the inverse of the information matrix and parameter vector respectively. Algorithm is characterized by the regressor matrix (stack of column regressor vectors) Q_k, update matrix S_k, signature matrix $D = \mathrm{diag}[1, -1]$ and the augmented output $\tilde{y}_k$, [24]. Rank two RLS provides quadratic complexity, tunable forgetting and window size, but is sensitive to ill-conditioning of the information matrix. Convergence of the algorithm can be proved using simplified models, [26]. The transient bound can be obtained on the Lyapunov function $V_k = \tilde{\theta}_k^T \Gamma_k^{-1} \tilde{\theta}_k$ for the algorithm, [26] as follows: $V_k \leq \rho^{\lfloor k/w \rfloor} V_0$, where $\rho = \lambda^w - c\lambda^w$, $0 < \rho < 1$ for persistently exciting regressor and $\xi_k = 0$.

4.3 Combined Algorithms & Noise Cancellation

Figure 1 shows that RLS algorithm provides better cancellation of the background noise than Kaczmarz algorithm in dominant UAV frequency extraction method. However, the intensity of the band for UAV is much lower for RLS method. This motivates using the RLS algorithm for acoustic noise cancellation, improving detection reliability by enhancing signal clarity for extracting dominant UAV frequencies. A few frequencies which are dominant for the background noise were included in optimization of the classifier (6). Extracted by RLS algorithm the signal which contains the main components of the background noise is subtracted from the measured signal, [26] (in other words the noise is canceled) for further processing by Kaczmarz algorithm and dominant UAV frequency extraction method described above. Hence, the noise is canceled by RLS algorithm and the intensity of UAV signal is emphasized by Kaczmarz method. Clearly, inclusion of noise cancellation improves signal quality and overall detection performance, see Figures 3(a) and 3(b). However, robustness problems may appear for non-stationary background noise with time varying dominant frequencies.

5. CONCLUSIONS

The increasing use of UAVs has created a growing need for effective detection systems, particularly in counterterrorism applications. Among the available sensing technologies, acoustic detection offers significant advantages due to its passive nature and low implementation cost. UAV propulsion mechanisms produce dominant acoustic components within a characteristic frequency band, allowing the extraction of informative spectral features for detection purposes. The total intensity in this band was modeled in this paper as a stochastic process consisting of the sum of normally distributed spectral amplitudes.
The effectiveness of the detection system is determined by the degree of separation between the statistical distributions associated with UAV signals and background noise. Accordingly, the detection problem was formulated as a hypothesis test between two competing models: background noise alone and background noise containing UAV activity. Improved classification performance was achieved through optimal frequency band selection and the refinement of amplitude estimation algorithms to minimize overlap between the corresponding distributions.
The findings of this study confirm that classifier performance is strongly influenced by the choice of algorithm. Accordingly, the Kaczmarz projection method was selected for signal amplitude estimation within the target frequency band. Moreover, the analysis established that the Kaczmarz method is a special case of the LMS algorithm that yields the maximum possible error reduction per iteration. The best detection result was achieved by the classifier that integrates RLS method for noise cancellation and Kaczmarz approach for dominant frequency extraction.
The proposed framework was validated using real-world acoustic recordings of UAVs and environmental background noise.

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