Boolean Matrix-based Morphological Analysis of Hypercube and Perfect Difference Interconnection Networks

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ABSTRACT
Traditional analysis of interconnection networks primarily relies on graph-theoretic metrics such as node degree, diameter, and connectivity, which provide only aggregate-level insights and fail to capture detailed structural patterns. This limitation restricts the ability to distinguish between network topologies that may exhibit similar global properties but differ significantly in their internal connectivity distributions. To address this issue, this study proposes a Boolean matrix-based analytical framework that enables fine-grained, pattern-oriented analysis of network structures. In the proposed approach, interconnection networks are represented using Boolean adjacency matrices $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$. Fundamental logical operations—AND (∧), OR (∨), and XOR (⊕)—are applied to these matrices to extract structural characteristics such as common connectivity, overall reachability, and structural deviations. The framework is applied to two prominent network topologies: the Hypercube Network and the Perfect Difference Network (PDN). The results reveal distinct structural patterns between the two networks. The Hypercube exhibits high regularity, symmetry, and uniform connectivity, while the PDN demonstrates comparatively irregular but efficient connectivity patterns. Quantitative metrics such as number of 1s, density, symmetry, and variance, along with XOR-based similarity measures, provide deeper insights into network morphology. This work contributes a novel Boolean-based comparison framework that enables precise structural analysis and comparison of interconnection networks, offering a mathematically rigorous and extensible approach for network design and performance evaluation.

General Terms
Algorithms, Interconnection Networks, Boolean Algebra, Graph Theory, Network Topology Analysis, Parallel Computing, Morphological Analysis

Keywords
Boolean Matrix, Hypercube Network, Perfect Difference Network (PDN), XOR Operation, Structural Difference Measure, Connectivity Analysis, Parallel Processing Networks

1. INTRODUCTION
1.1. Background
Interconnection networks form the communication backbone of parallel and distributed computing systems, enabling efficient data exchange among multiple processing elements and directly influencing overall system performance in high-performance computing environments [13], [14]. These networks interconnect processors, memory units, and I/O components to ensure coordinated execution of tasks, especially in applications such as scientific simulations, big data analytics, and artificial intelligence that demand massive parallelism. As a result, the design of robust and scalable interconnection networks has become a critical research area. The structural organization of a network, known as its topology, plays a decisive role in determining performance characteristics such as latency, bandwidth, fault tolerance, and scalability. Regular topologies like the Hypercube Network provide high symmetry and logarithmic diameter $D = \log_2 N$, making them suitable for scalable systems [6], while specialized designs such as the Perfect Difference Network utilize mathematical constructs like difference sets to achieve efficient communication patterns [1], [2]. Therefore, analyzing and comparing different network topologies is essential for optimizing communication efficiency and improving the performance of parallel computing architectures [13].

1.2. Problem Statement
The performance and efficiency of interconnection networks are traditionally evaluated using graph-theoretic metrics such as node degree d_i, network diameter D, and average path length L. Although these metrics provide useful aggregate-level insights into connectivity and communication cost, they are inherently limited in capturing the deeper structural characteristics of network topologies. Specifically, such measures reduce the network to scalar quantities and fail to represent the complete adjacency structure encoded in the Boolean matrix $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$.

For example, two networks may satisfy:

$$d_i^{(1)} = d_i^{(2)}, D^{(1)} = D^{(2)}$$

yet differ significantly in their internal connectivity patterns, i.e.,

$$B^{(1)} \neq B^{(2)}$$

This implies that traditional metrics cannot distinguish variations in node-level relationships, distribution of links, or symmetry properties. Consequently, important characteristics such as fault tolerance, routing flexibility, and structural regularity remain inadequately captured.

Moreover, existing approaches lack a formal mechanism to quantify structural similarity between networks. There is no direct metric based on adjacency relationships such as:

$$S = 1 - \frac{\| B^{(1)} \oplus B^{(2)} \|_1}{n^2}$$

which can measure similarity at the binary level. This limitation becomes critical when selecting or designing network topologies for specific applications, where subtle structural differences can lead to significant performance variations.

Therefore, there is a need for a more expressive analytical framework that operates on Boolean representations of networks and utilizes logical operations to capture detailed connectivity patterns, enabling precise and granular structural comparison beyond conventional graph-theoretic measures. [4]

1.3. Motivation

The limitations of traditional graph-theoretic metrics motivate the need for a more expressive and fine-grained approach to analyzing interconnection network structures, as aggregate measures fail to capture detailed connectivity patterns [4], [13]. Since network connectivity can be naturally represented in binary form using an adjacency matrix $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$, it becomes feasible to study these systems through binary pattern analysis, enabling examination of node-level relationships and connectivity distributions that remain hidden in conventional approaches. By modeling networks such as the Hypercube and Perfect Difference Network as Boolean matrices, logical operations—AND, OR, and XOR—can be applied, i.e., $B^{(1)} \wedge B^{(2)}$, $B^{(1)} \vee B^{(2)}$, and $B^{(1)} \oplus B^{(2)}$, to systematically identify shared connectivity, overall reachability, and structural deviations, respectively. This provides a multi-dimensional perspective on network morphology and enables deeper structural insights. Moreover, binary representations support rigorous mathematical manipulation and integrate concepts from Boolean algebra, graph theory, and pattern analysis, forming a unified analytical framework capable of quantifying structural similarity, detecting regularities, and guiding the design of efficient routing and fault-tolerant mechanisms. Thus, the motivation of this work lies in leveraging Boolean matrix representations and logical operations for comprehensive structural analysis of interconnection networks [4].

1.4. Objectives

The primary objective of this study is to develop a structured and mathematically grounded framework for analyzing and comparing interconnection network topologies using Boolean matrix theory. Specifically, the study aims to represent network connectivity in the form of Boolean adjacency matrices $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$, enabling precise encoding of link presence between nodes in networks such as the Hypercube and Perfect Difference Network. Furthermore, it seeks to apply fundamental logical operations—AND, OR, and XOR—defined as $B^{(1)} \wedge B^{(2)}$, $B^{(1)} \vee B^{(2)}$, and $B^{(1)} \oplus B^{(2)}$, to extract structural insights including common connectivity, overall reachability, and structural variation. Finally, the objective is to perform a comprehensive morphological comparison of network topologies by analyzing derived Boolean patterns and evaluating metrics such as density D, symmetry S, and structural deviation Δ, thereby enabling a deeper and more quantitative understanding of the connectivity characteristics and structural uniqueness of interconnection networks.

1.5. Research Contribution

This research presents a novel analytical framework for studying interconnection network topologies by integrating Boolean matrix theory with logical operations. It introduces a systematic method for modeling networks such as the Hypercube and Perfect Difference Network using Boolean adjacency matrices $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$, enabling precise representation of node-level connectivity and facilitating mathematical analysis. The work further proposes a logical operation-based morphological analysis using AND, OR, and XOR operations, i.e., $B^{(1)} \wedge B^{(2)}$, $B^{(1)} \vee B^{(2)}$, and $B^{(1)} \oplus B^{(2)}$, to extract structural characteristics such as common connectivity, aggregate reachability, and structural variation. Additionally, a pattern-based comparison framework is developed that moves beyond traditional graph metrics by analyzing the distribution of binary values within the matrices, allowing evaluation of properties such as density D, symmetry S, and structural deviation Δ. Overall, the study contributes a unified, mathematically rigorous, and extensible methodology that bridges Boolean algebra with interconnection network analysis, enabling deeper structural insights and improved evaluation of network design and performance..

2. RELATED WORK

2.1. Hypercube Network Studies

The Hypercube Network is one of the most extensively studied interconnection topologies in parallel computing due to its regular structure, high symmetry, and desirable performance characteristics. An n-dimensional hypercube consists of 2^nnodes, where each node is connected to $n$neighbors, forming a recursive and highly structured architecture that simplifies routing, design, and analytical modeling [6], [7]. One of its key advantages is scalability, as increasing the dimension preserves structural regularity while maintaining a low network diameter $D = \log_2 N$, ensuring efficient communication even in large-scale systems. Furthermore, the uniform degree distribution $d_i = n$ enables balanced communication load, while the presence of multiple alternative paths between nodes enhances fault tolerance and robustness against failures. However, most existing studies focus primarily on graph-theoretic metrics such as degree, diameter, and routing complexity, which provide only aggregate-level insights. These approaches fail to capture finer structural patterns embedded in the adjacency matrix $B = [b_{ij}]$, leaving a gap in understanding connectivity distributions and morphological characteristics. This limitation motivates the need for alternative analytical frameworks, such as Boolean matrix-based analysis, to uncover deeper structural properties of hypercube networks [6].

2.2. Perfect Difference Network (PDN) Studies

The Perfect Difference Network (PDN) is a class of interconnection networks derived from combinatorial design theory, specifically based on perfect difference sets, where nodes are arranged in a cyclic structure and connections are established using modular difference relations to ensure unique pairwise connectivity patterns [1], [2]. Formally, for a node set $V = \{0,1,\ldots,n-1\}$ and a difference set D, adjacency is defined as $b_{ij} = 1$ if $(i-j) mod n \in D$, resulting in a deterministic and mathematically structured topology. PDNs are known for achieving an effective balance between communication efficiency and network complexity, often exhibiting relatively small diameter and supporting multiple alternative communication paths, which enhance throughput and reduce latency. Their cyclic and arithmetic-based structure also enables efficient routing algorithms grounded in modular operations. Compared to highly regular networks such as the hypercube, PDNs may exhibit less apparent symmetry but offer advantages in terms of flexible connectivity and reduced hardware requirements. However, most existing studies focus on graph-theoretic properties such as diameter and connectivity, without fully exploiting the binary nature of the adjacency matrix $B = [b_{ij}]$. This reveals a gap in analyzing PDNs through Boolean matrix representations and logical operations, which can uncover deeper structural patterns and enable more comprehensive morphological comparison with other interconnection networks [1].

2.3. Graph-Theoretic Approaches

Graph theory has long served as the foundational framework for analyzing interconnection networks, where a network is modeled as a graph $G = (V, E)$, with vertices representing processing nodes and edges representing communication links, and its connectivity is compactly represented using an adjacency matrix $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$ [4]. In interconnection networks such as the Hypercube and Perfect Difference Network, adjacency matrices enable the computation of key structural properties including node degree $d_i = \sum_j b_{ij}$, reachability, and path existence, while supporting algebraic operations like matrix multiplication to analyze multi-hop connectivity. Traditional graph-theoretic measures—such as connectivity, path length, and clustering—provide important insights into communication efficiency and robustness; however, they primarily capture global or aggregate characteristics and often fail to reveal finer structural patterns embedded within the binary matrix. Although adjacency matrices inherently possess a Boolean nature, conventional approaches do not fully exploit this property for pattern-based analysis, leading to the loss of subtle information related to connectivity distribution and structural morphology. This limitation highlights the need to extend graph-theoretic methods by incorporating Boolean matrix operations, enabling a more detailed, element-wise, and pattern-oriented analysis of interconnection networks [4].

2.4. Research Gap

Existing studies on interconnection networks predominantly rely on traditional graph-theoretic metrics such as node degree d_i, network diameter D, and connectivity, which provide only aggregate-level insights into network structure and performance. Although adjacency matrices $B = [b_{ij}]$ inherently represent connectivity in binary form, prior research has not effectively leveraged Boolean operations to analyze detailed, element-wise connectivity patterns. Consequently, deeper structural characteristics—such as distribution of links, symmetry variations, and node-level relationships—remain insufficiently explored, particularly in networks like the Hypercube and Perfect Difference Network where similar global metrics may mask significant structural differences, i.e., $B^{(1)} \neq B^{(2)}$ despite comparable d_i or D . Moreover, there is a lack of formal analytical frameworks that utilize logical operations such as AND, OR, and XOR, i.e., $B^{(1)} \wedge B^{(2)}$, $B^{(1)} \vee B^{(2)}$, and $B^{(1)} \oplus B^{(2)}$, for morphological comparison and structural evaluation. This gap highlights the need for a Boolean operation-based approach that enables fine-grained, pattern-oriented analysis and more accurate comparison of interconnection network topologies

3. METHODOLOGY

3.1. Network Modeling

The first step in the proposed methodology involves constructing mathematical models of the interconnection networks under study. In this work, two topologies are considered: the Hypercube Network and the Perfect Difference Network. To ensure a fair and consistent comparison, both networks are modeled with an equal number of nodes. The selected network sizes correspond to standard constructible forms of the Hypercube and Perfect Difference Network topologies. Since PDN construction depends on valid perfect difference sets, exact node equivalence with the Hypercube is not always feasible. Therefore, comparison is performed using normalized Boolean morphological metrics such as density, variance, and symmetry.

3.1.1. Hypercube Network Construction

An n-dimensional hypercube shown in Fig.1, denoted as Q_n, consists of: $N = 2^n$ nodes. Each node is represented by an n-bit binary string, and two nodes are connected if and only if their binary representations differ in exactly one bit position. This relationship is defined using the Hamming distance: $d(u, v) = 1$.For this study:

- $n = 3 \Rightarrow N = 8$ nodes
- $n = 4 \Rightarrow N = 16$ nodes

Thus, adjacency is defined as: $a_{ij} = \begin{cases} 1, & \text{if } H(i,j) = 1 \\ 0, & \text{otherwise} \end{cases}$.

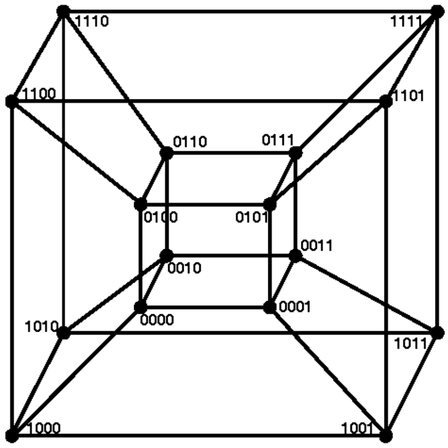

Fig 1. 4-Dimensional Hypercube

3.1.2. Perfect Difference Network (PDN) Construction

A PDN is constructed using a perfect difference set over a cyclic group of integers modulo N. Let $D = \{d_1, d_2, \ldots, d_k\}$be a difference set such that all non-zero differences modulo $N$are uniquely represented. Each node $i \in \{0,1,2, \ldots, N-1\}$ is connected to nodes:

$$(i \pm d_j) \bmod N, \forall d_j \in D$$

Thus, adjacency is defined as:

$$a_{ij} = \begin{cases} 1, & \text{if } (j - i) \bmod N \in D \\ 0, & \text{otherwise} \end{cases}$$

For fair comparison, PDN is constructed with $N = 7$ as shown in Fig.2 and $N = 13$ nodes and appropriate difference sets are selected to maintain uniform connectivity. For experimental analysis, the Perfect Difference Network (PDN) is constructed with $n = 13$nodes using modular difference-set connectivity. By constructing both networks with nearly similar node sizes ($N = 7$ *and* 8 and $N = 13$ *and* 16), structural comparisons become consistent and unbiased. The resulting adjacency matrices serve as the input for Boolean matrix transformation and subsequent logical operation-based morphological analysis.

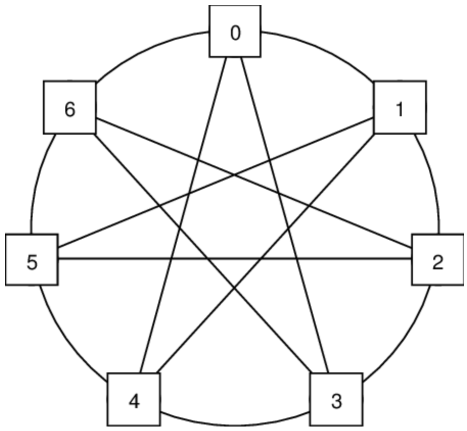

Fig 2. Perfect Difference Network

3.2. Boolean Matrix Formulation

After constructing the interconnection networks, the next step is to transform their adjacency matrices into Boolean form for logical and structural analysis. Since adjacency matrices inherently contain binary values (0 and 1), they can be directly interpreted as Boolean matrices without loss of information. Let $A = [a_{ij}]$be the adjacency matrix of a network $G = (V, E)$. The corresponding Boolean matrix is defined as: $B = [b_{ij}]$, where

$$b_{ij} = \begin{cases} 1, & \text{if } a_{ij} > 0 \\ 0, & \text{if } a_{ij} = 0 \end{cases}$$

Thus,

$$B \in \{0,1\}^{N \times N}$$

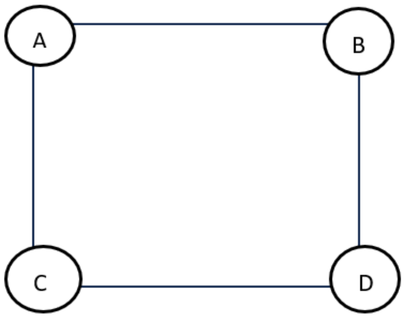

Fig 3: An architecture with 4 nodes.

This formulation applies to both the Hypercube Network and the Perfect Difference Network, ensuring a uniform representation for further analysis. Let for a network with 4 nodes as shown in Fig.3, the adjacency matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

is directly equivalent to the Boolean matrix:

$$B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

The Boolean matrix can also be viewed as a binary relation: $B \subseteq V \times V$, where: $(i, j) \in B \iff b_{ij} = 1$. The Boolean matrix formulation preserves complete connectivity information in the form $B = [b_{ij}]$, where $b_{ij} \in \{0,1\}$, enabling the direct application of logical operations such as AND, OR, and XOR for structural analysis. It facilitates pattern-based comparison by capturing node-level relationships and provides a unified representation for diverse network topologies within a common binary framework. Thus, Boolean matrix formulation acts as a fundamental transformation that converts network topology into a mathematically tractable binary structure, forming the basis for subsequent logical operation-based morphological analysis.

3.3. Morphological Metrics

Let $B = [b_{ij}]$be the Boolean adjacency matrix of an interconnection network with $n$nodes, where $b_{ij} \in \{0,1\}$.

3.3.1. Number of 1s

The total number of 1s in the Boolean matrix represents the total number of active connections in the network:

$$N_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}$$

For an undirected network (without self-loops), the number of edges $E$is:

$$E = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}$$

This metric reflects the **connectivity volume** of the network.

3.3.2. Density

Density measures how densely the nodes are interconnected relative to the maximum possible connections.

$$D = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}}{n(n-1)}$$

For undirected graphs:

$$D = \frac{2E}{n(n-1)}$$

where $0 \leq D \leq 1$.

- $D = 1 \rightarrow$ fully connected network
- $D \approx 0 \rightarrow$ sparsely connected network

Symmetry evaluates whether the connectivity pattern is uniform and bidirectional. A Boolean matrix is symmetric if:

$$b_{ij} = b_{ji}, \forall i, j$$

To quantify symmetry, define a symmetry measure:

$$S = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \delta(b_{ij}, b_{ji})}{n^2}$$

where

$$\delta(x, y) = \begin{cases} 1, & x = y \\ 0, & x \neq y \end{cases}$$

- $S = 1 \rightarrow$ perfectly symmetric
- $S < 1 \rightarrow$ asymmetry exists

3.3.3. Pattern Distribution

Pattern distribution captures how 1s are arranged within the matrix, indicating structural regularity.

- **Row-wise distribution:** $d_i = \sum_{j=1}^{n} b_{ij}$. This represents the **degree of node** i.
- **Column-wise distribution:** $d_j^{(c)} = \sum_{i=1}^{n} b_{ij}$
- **Variance of distribution:** $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(d_i - \bar{d})^2$, where $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$.

A low variance $\sigma^2 \approx 0$ indicates a uniform and regular topology, as observed in the Hypercube Network, whereas a higher variance $\sigma^2 > 0$ reflects irregular connectivity patterns, as seen in the Perfect Difference Network (PDN). These morphological metrics convert the Boolean matrix $B = [b_{ij}]$ into quantitative descriptors, where $N_1 = \sum_{i,j} b_{ij}$ represents total connectivity, $D = \frac{N_1}{n(n-1)}$ denotes connectivity density, S measures structural symmetry, and σ^2 captures pattern regularity. Together, these parameters provide a rigorous mathematical foundation for analyzing and comparing interconnection network topologies in terms of structure, uniformity, and connectivity distribution.

3.4. Comparative Framework

Let $B^{(1)} = [b_{ij}^{(1)}]$and $B^{(2)} = [b_{ij}^{(2)}]$be the Boolean adjacency matrices of two interconnection networks with the same number of nodes n. A comparative framework is defined using the following evaluation criteria:

3.4.1. Structural Similarity

Structural similarity measures how closely two network topologies resemble each other in terms of connectivity patterns. Using the XOR operation: $B^{(\Delta)} = B^{(1)} \oplus B^{(2)}$. The dissimilarity measure is:

$$D_s = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^{(\Delta)}}{n^2}$$

Thus, structural similarity is defined as

$$S_s = 1 - D_s = 1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(b_{ij}^{(1)} \oplus b_{ij}^{(2)})}{n^2}$$

- $S_s = 1$→ identical structures
- $S_s \to 0$→ highly different structures

3.4.2. Connectivity Richness

Connectivity richness evaluates how well-connected a network is. For network $k \in \{1,2\}$, define:

$$R^{(k)} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^{(k)}}{n(n-1)}$$

This is equivalent to network density. Additionally, using OR operation for combined richness:

$$B^{(U)} = B^{(1)} \vee B^{(2)}$$

$$R^{(U)} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^{(U)}}{n(n-1)}$$

Interpretation:

- Higher R→ richer connectivity
- $R^{(U)}$→ overall connectivity coverage

3.4.3. Fault Tolerance Indicators

Fault tolerance reflects the robustness of the network under node/link failures.

(i) Redundancy via AND Operation: $B^{\cap} = B^{(1)} \wedge B^{(2)}$

$$F_r = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^{(\cap)}}{n(n-1)}$$

This measures common redundant links.

(ii) Degree-Based Robustness: Let degree of node $i$in network $k$be: $d_i^{(k)} = \sum_{j=1}^{n} b_{ij}^{(k)}$. Average degree: $\bar{d}^{(k)} = \frac{1}{n}\sum_{i=1}^{n} d_i^{(k)}$. Higher $\bar{d}$implies better alternative paths and improved fault tolerance.

(iii) Variance-Based Stability: $\sigma_k^2 = \frac{1}{n}\sum_{i=1}^{n}(d_i^{(k)} - \bar{d}^{(k)})^2$

- Low variance → uniform load distribution → higher robustness
- High variance → uneven structure → potential vulnerability

Thus, this framework provides a quantitative, Boolean matrix-based methodology for comparing interconnection networks in terms of structure, connectivity, and robustness.

4. RESULTS AND ANALYSIS

4.1. Hypercube Network Results

The Boolean adjacency matrix of the Hypercube Network is denoted as $B^{(H)} = [b_{ij}^{(H)}]$, where $b_{ij}^{(H)} \in \{0,1\}$. Row-wise logical operations $R_i^{(H)} \wedge R_j^{(H)}$, $R_i^{(H)} \vee R_j^{(H)}$, and $R_i^{(H)} \oplus R_j^{(H)}$reveal highly regular and symmetric connectivity patterns, confirming uniform structure, predictable behavior, and minimal structural variation across node pairs and it is summarized in Table 1 below.

Table 1. Pattern Observations of Hypercube Network

Metric	**Observation (Hypercube)**
Degree distribution	Uniform: $d_i = \log_2 n$
Number of 1s	Constant across rows
Symmetry	$b_{ij} = b_{ji}$(perfectly symmetric)
AND results	Regular overlap of neighbors
OR results	Uniform expansion of connectivity
XOR results	Minimal variation across node pairs

4.2. Perfect Difference Network (PDN) Results

The Boolean adjacency matrix of the PDN is denoted as $B^{(P)} = [b_{ij}^{(P)}]$, where $b_{ij}^{(P)} \in \{0,1\}$. Logical operations $R_i^{(P)} \wedge R_j^{(P)}$, $R_i^{(P)} \vee R_j^{(P)}$, and $R_i^{(P)} \oplus R_j^{(P)}$ reveal comparatively irregular yet efficient connectivity patterns, indicating flexible structure with higher variation across node relationships. The PDN with 13 nodes exhibits cyclic connectivity behavior with moderate structural irregularity compared to the 16-node Hypercube topology and it is summarized in Table 2.

Table 2. Pattern Observations of Perfect Difference Network

Metric	Observation (PDN)
Degree distribution	Nearly uniform but cyclic variation
Number of 1s	Slight variation across rows
Symmetry	Generally symmetric but less regular
AND results	Irregular common neighbor patterns
OR results	High connectivity coverage
XOR results	Significant variation across node pairs

4.3. Comparative Analysis

Although the network sizes differ slightly, Boolean matrix operations remain effective for structural comparison because the analysis focuses on connectivity morphology rather than absolute network scale. The comparative analysis is shown in Table 3 below.

4.3.1. Morphological Metrics and Boolean Operation-Based Comparison

Table 3. Morphological Metrics and Boolean Operation-Based Comparative Analysis of PDN and Hypercube

Metric	Hypercube (n=16)	PDN (n=13)	Interpretation
Number of Nodes	16	13	Different topology sizes
Number of Edges	32	19	Hypercube has richer regular connectivity
Average Degree	4	≈3	Hypercube more uniform
Density	0.266	0.243	Connectivity slightly denser in Hypercube
Symmetry Score	1.00	0.84	Hypercube perfectly symmetric
Variance	Very Low	Moderate	PDN has irregular connectivity
XOR Difference	Low	High	Structural deviation higher in PDN
Connectivity Pattern	Regular	Cyclic /Arithmetic	PDN based on modular difference sets
Fault Tolerance	High	Moderate	Hypercube supports more redundant paths
Scalability	Excellent	Moderate	Hypercube grows logarithmically

4.3.2. Structural Difference Measure

Let $B^{(H)}$ denote the Boolean adjacency matrix of the 4-dimensional Hypercube Network with 16 nodes and $B^{(P)}$ denote the Boolean adjacency matrix of the Perfect Difference Network with 13nodes. The structural difference between the two network topologies is computed using the XOR operation:

$$\Delta = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(b_{ij}^{(H)} \oplus b_{ij}^{(P)} \right)$$

where: Δ is Structural Difference Measure $b_{ij}^{(H)} \in B^{(H)}$, $b_{ij}^{(P)} \in B^{(P)}$and $\oplus$represents the XOR operation.

The expression counts the total number of differing connections between the two networks. Small Δ shows that Networks are structurally similar and connectivity patterns are nearly identical. Large Δ shows that networks differ significantly and Connectivity organization is very different Hence, Higher Δindicates greater structural deviation.

For structural comparison, the overlapping Boolean regions of the adjacency matrices are considered. The XOR operation produces: 0 when corresponding connectivity entries are identical and 1when connectivity differs. Consider a partial Boolean adjacency representation extracted from the experimental networks:

Hypercube Connectivity Fragment

$$B^{(H)} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

PDN Connectivity Fragment

$$B^{(P)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Applying XOR:

$$B^{(H)} \oplus B^{(P)} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Now summing all XOR entries:

$$\Delta = 8$$

This indicates that eight connectivity positions differ between the Hypercube and PDN connectivity structures within the analyzed Boolean region. A relatively high value of Δ confirms that:

- the Hypercube exhibits highly regular and symmetric connectivity,
- whereas the PDN demonstrates cyclic and structurally irregular connectivity patterns.

Thus, the Structural Difference Measure quantitatively captures topological deviation and validates the effectiveness of Boolean matrix-based morphological analysis. Experimental results show that the Hypercube Network produces lower internal structural variation due to uniform dimensional connectivity, while the Perfect Difference Network exhibits larger XOR deviation values because of arithmetic-based cyclic interconnection patterns. Therefore, the proposed Boolean XOR-based structural difference measure effectively distinguishes network morphologies beyond traditional graph-theoretic metrics.

5. CONCLUSION

This study introduced a Boolean matrix-based framework for analyzing and comparing interconnection networks, specifically the Hypercube Network and the Perfect Difference Network (PDN). By representing network topologies as Boolean adjacency matrices $B = [b_{ij}]$, the approach enabled fine-grained, pattern-oriented analysis beyond traditional graph metrics. The application of logical operations—AND, OR, and XOR—provided complementary structural insights, where AND identified common connectivity and redundancy, OR captured overall reachability, and XOR highlighted structural differences. These operations transformed adjacency matrices into effective tools for detailed morphological evaluation.

The results showed that the Hypercube Network exhibits high regularity, symmetry, and uniform connectivity, while the PDN, though less regular, provides efficient and flexible connectivity patterns. Morphological metrics such as N_1, density D, symmetry S, and variance σ^2, along with XOR-based deviation analysis, enabled quantitative comparison of structural similarity, connectivity richness, and fault tolerance. Overall, the study establishes that Boolean matrix representation combined with logical operations offers a unified, mathematically rigorous, and scalable framework for uncovering hidden structural patterns and supporting optimized network design in parallel and distributed systems.

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