Availability and Dependability Analysis of Active-Passive Cluster Systems using Semi-Markov Model with Parametric Study

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ABSTRACT
Active-passive cluster systems are commonly used in distributed computing to improve system availability and fault tolerance. In this work, Semi-Markov model is used to analyze the availability and dependability of the system. A state transition diagram is used to represent the state behavior which includes active unit failure, standby unit failure, coverage, and failure detection. System availability is evaluated by calculating the steady-state probabilities. The impact of the active unit failure rate on availability is analyzed using parametric evaluation and graphical results. The results show us that the availability decreases when the failure rate increases, which highlights the importance of component reliability. The model presented can help in the design and evaluation of highly available cluster system.

General Terms
Availability Analysis, Fault Tolerance, Distributed Systems, Reliability Engineering.

Keywords
Cluster Systems, Dependability, Semi-Markov Model, High Availability Systems, Failure Rate.

1. INTRODUCTION
In today's distributed computing environments, availability and system reliability are improved using the cluster systems [1], [3], [7]. Cluster architecture is used in many enterprise applications, such as database servers, cloud platforms, and distributed storage systems to ensure that services can continue running even if some components fail [1], [4]. Even though redundancy and failover mechanisms are used, system failures cannot be completely avoided because hardware can fail, software errors may occur, and network problems can also affect the system. Therefore, studying the dependability of such systems has become an important research area.

Dependability is commonly described in terms of availability, reliability, safety and maintainability [9], [11]. Among these factors, availability is often considered the most important for cluster systems because service interruption can result in data loss, financial loss, and system downtime. In many practical systems, an active-passive cluster configuration is used, where a standby unit takes over operation when the active unit fails [3], [7]. The overall system performance and dependability depend on several factors, including failure rates, repair rates, failure detection mechanisms and coverage probabilities.

Markov and Semi-Markov processes which use the mathematical modeling technique are widely used to analyze reliability and availability of repairable systems [5], [8], [9]. In traditional Markov models, transition times are assumed to follow exponential distributions. Semi-Markov models, however, are more flexible because they allow more general time distributions for repair and detection processes, which makes them more suitable for representing realistic system behavior and evaluating system availability more accurately [5], [9].

In many previous studies, Markov and Semi-Markov were used to analyze the availability of cluster systems [3], [7], [20]. Many of these studies focus mainly on model formulation and steady-state availability computation. This paper employs a Semi-Markov model to analyze the availability and dependability of an active-passive cluster system. A state transition diagram is used to describe the system behavior, including active unit failure, standby unit failure detection, and coverage conditions. The steady-state probabilities are calculated from the model and used to determine system availability. By varying the failure rate of the active unit, its effect on system availability is studied and results are presented graphically. The analysis provides insight into system dependability behavior and can be useful in the design of high availability distributed systems.

The remainder of this paper is organized as follows. Section 2 presents related work. Section 3 describes the system model and assumptions. Section 4 contains the Semi-Markov model and mathematical analysis. Section 5 shows results and discussion. At last, Section 6 concludes the paper and outlines the future work.

2. RELATED WORK
Dependability and availability of computer systems have been studied for many years in the areas of reliability engineering and distributed computing [9],[11]. High availability is very important for mission-critical application since system failures can cause service interruption and sometimes data loss. To study the reliability and availability of repairable system different modeling techniques such as Markov models, Semi-Markov models, Stochastic Petri nets, and fault tree analysis are often used [10], [11], [17].

For analysis of availability and reliability, Markov models are often used as they are mathematically simple and easier to handle compared to many other models. Continuous-time Markov chains are used to represent system failures, repair processes, and recovery behavior in distributed and cluster systems [3], [7], [20]. A key limitation of Markov models is that they assume transition times follow exponential distributions, which may not always accurately represent real system behavior.

Semi-Markov models extending the Markov models allows more general distributions for repair time and detection time [5], [9]. So, Semi-Markov are more suitable for modeling repairable systems in real situations. These models have been applied in reliability and availability analysis of computer systems, communication systems, and cluster-based architectures where repair and recovery times do not strictly follow exponential distributions.

Availability modeling of cluster systems, particularly active-standby and active-passive configurations has also been studied by several researchers using stochastic models. These studies evaluate system availability and dependability under different failure and repair conditions.

Using stochastic Petri nets and fault tree model also several studies have been made on availability modeling of repairable systems by several researchers. These approaches are useful for complex systems with multiple failure modes and repair process. As system need to be modeled explicitly with state transitions and repair behavior, Markov and Semi-Markov models are preferred due to their mathematical tractability and ability to compute steady-state availability. Availability analysis in the cluster system environment is important for designing fault-tolerant systems and minimizing system downtime.

In this paper, availability and dependability analysis of an active-passive cluster is performed using a Semi-Markov model. The system model includes failure coverage, standby unit failure detection and repair process. The steady-state availability of the system is evaluated and the effect of active unit failure rate on system availability is analyzed.

3. SYSTEM MODEL AND ASSUMPTIONS

3.1 System Description

This study considers an active-passive cluster system to improve system availability and fault tolerance. The system consists of two units, active unit and standby unit. During normal operation, service is provided by active unit, the standby unit stays ready to take over if the active unit fails.

Controller (broker) continuously monitors the system and handles the failover process when a failure occurs. Whenever the active unit fails, the broker tries to transfer the service to the standby unit. The success of switching depends on the coverage probability related to the active unit failure.

The standby unit may also fail while the active unit is still operational. In some cases, the standby unit failure is detected immediately, while in other cases the failure remains latent and is detected after a certain time delay. The system model takes into account these different failure and detection scenarios.

In our study, the system behavior in the given conditions is described using Semi-Markov model. Repair and detection times do not always follow exponential distributions. Semi-Markov model provides a more realistic representation of the system.

3.2 Model Parameters

The following parameters are used in the system model:

Table 1. Model Parameters

Symbol	Description
λ	Failure rate of active unit
λ_s	Failure rate of standby unit
M	Repair/restoration rate of failed unit
C	Coverage probability of active unit failure
c_s	Coverage probability of standby unit failure
T	Time interval for detection of standby unit failure
U(T)	Probability of detecting standby unit failure within time T
π_i	Steady-state probability of system being in state i.

These parameters describe the failover behavior, repair behavior, and detection behavior of the system components.

3.3 Model Assumptions

Following are the assumptions made while developing the system model.

1. The system consists of one active unit and one standby unit in active-passive configuration.
2. Failure times of active and standby units follow exponential distribution with failure rates λ and λs respectively.
3. Failed units are repaired with restoration rate μ.
4. When the active unit fails, the system attempts to switch service to the standby unit with coverage probability c.
5. The standby unit failure may be detected immediately with probability cs or may remain undetected for some time.
6. The probability of detecting standby unit failure within time T is represented by U(T).
7. After repair, failed units are statistically independent.
8. Failures and repairs are statistically independent.
9. The system is considered operational as long as service is available to users.

Using these assumption Semi-Markov transition model is constructed.

3.4 System States

Six states are used to draw the system model. These states represent all possible operating and failure condition of active and standby units. Below are the state descriptions being used.

Table 2. System States

State	Description
1.	Both active and standby units are operational.
2.	Active unit fails and coverage mechanism fails.
3.	Active unit fails and standby unit successfully takes over the service.
4.	Standby unit fails while active unit is operational and t failure is detected.
5.	Standby unit fails but the failure is not detected.
6.	System failure state.

3.5 State Transition Diagram

Figure 1 represents the state transition diagram for the active-passive cluster system. Transitions between states occur due to component failures, repair processes, and failure detection mechanisms. The active unit fails with failure rate λ, and the standby unit fails with failure rate λ_s. Failed units are repaired with restoration rate μ. The coverage probabilities for active and standby unit failures are represented by c and c_s, respectively. The standby unit failure detection probability is represented by U(T).

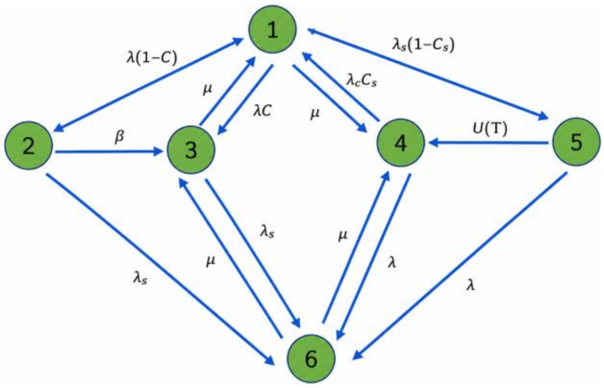

Fig 1: State Transition Diagram of the Active-Passive Cluster System

Similar state transition models have been used in availability analysis of active-standby and active-passive cluster systems in previous studies. In this work, the model is used to analyze system availability and dependability and to study the impact of failure rate on system availability [3], [7], [20].

3.6 Availability Definition

If the service is available to users, then system is considered operational. From the state diagram, the system is operational in state 1,3,4,5 and failed in 2 and 6. So the steady-state availability of the system is given by

A = $\pi_1 + \pi_3 + \pi_4 + \pi_5$ --------(1)

Where πi represents the steady-state probability of th system being in state i.

4. SEMI-MARKOV MODEL AND MATHEMATICAL ANALYSIS

The active-passive cluster system described in Section 3 is analyzed using the Semi-Markov model. The Semi-Markov approach is used because the routine diagnostic used to detect latent standby unit failure introduces a non-exponential transition mechanism. While failure and repair times are assumed to be exponentially distributed, the diagnostic interval does not satisfy the memoryless property of a continuous-time Markov chain. Therefore, the system is modeled using a Semi-Markov process [5], [9].

4.1 Steady-State Balance Equations

Let Pi denote the steady-state probability of the system being in state i, where i=1,2,…,6. Based on the state transition diagram show in figure 1, the steady-state balance equations are obtained by equating the probability flow into and out of each state.

$\mu P_3 + \mu P_4 = \lambda(1-c)P_1 + \lambda c P_1 + \lambda_s c_s P_1 + \lambda_s(1-c_s)P_1$ ---(1)

$\lambda(1-c)P_1 = (\beta + \lambda_s)P_2$-------------------------(2)

$\lambda c P_1 + \beta P_2 + \mu P_6 = \mu P_3 + \lambda_s P_3$----------------(3)

$\lambda_s c_s P_1 + \frac{2}{T}P_5 + \mu P_6 = (\mu + \lambda)P_4$-------------(4)

$\lambda_s(1-c_s)P_1 = \left(\frac{2}{T} + \lambda\right)P_5$----------------------(5)

$\lambda_s P_2 + \lambda_{s\backslash}P_3 + \lambda P_4 + \lambda P_5 = 2\mu P_6$--------------(6)

4.2 Normalization Condition

The steady-state probabilities satisfy the normalization condition:

$$\sum_{i=1}^{n} P_i = 1$$

Solving the balance equations together with the normalization condition gives the steady-state probabilities of all system states.

4.3 Semi-Markov Formulation

Since the routine diagnostic interval does not follow exponential distribution, the system is modeled using a Semi-Markov process. The Semi-Markov process is described by an embedded discrete-time Markov chain and the mean sojourn times in each state.

Let P be the transition probability matrix of the embedded discrete-time Markov chain and Hi(t) be the sojourn time distribution in state i. The key transition probabilities are:

$H_1 = 1 - e^{-(\lambda+\lambda_s)t}$ -------------(1)

$H_2 = 1 - e^{-(\beta+\lambda_s)t}$ -----------------(2)

$H_3 = 1 - e^{-(\lambda_s+\mu)}$ ------------------(3)

$H_4 = 1 - e^{-(\lambda+\mu)t}$ ------------------(4)

$H_5 = \begin{Bmatrix} 1-\left[1-\frac{t}{T}\right]e^{-\lambda t}, t<T \\ 1, t>T \end{Bmatrix}$---------(5)

$H_6 = 1 - e^{2\mu t}$ ----------------------(6)

4.4 Mean Sojourn Times

The mean sojourn time in each state is given by:

$$h_i = \int_0^{\infty}(1 - H_i(t))dt$$

$h_1 = \frac{1}{\lambda+\lambda_s}$ ---------------- (1)

$h_2 = \frac{1}{\beta+\lambda_s}$ ---------------- (2)

$h_3 = \frac{1}{\lambda_s+\mu}$ ---------------- (3)

$h_4 = \frac{1}{\lambda+\mu}$ --------------------------------- (4)

$h_5 = \frac{1}{\lambda} - \frac{1}{T\lambda^2}\left(1 - e^{-\lambda T}\right)$ ---------------- (5)

$h_6 = \frac{1}{2\mu}$ ------------------------------------- (6)

4.5 Steady-State Probabilities of Semi-Markov Chain

The steady-state probabilities of the Semi-Markov chain are given by

$$\pi_i = \frac{v_i h_i}{\sum_j v_j h_j}$$

where vi represents the steady-state probability of the embedded discrete-time Markov chain and hi represents the mean sojourn time in state i.

4.6 Availability Expression

From the system state definitions, the system is operational in states 1,3,4 and 5. Therefore, the steady-state availability of the system is given by

$A = \pi_1 + \pi_3 + \pi_4 + \pi_5$

This availability expression is used in the next section to evaluate the effect of system parameters on overall system availability and dependability.

5. RESULTS AND DISCUSSION

5.1 Parameter Values

To evaluate the system availability, reasonable values of system parameters are assumed. The parameter values used in the analysis are shown in Table 3 below:

Table 3. System Parameter Values

Parameter	Value
Coverage probability (c)	0.9
Standby coverage probability (cs)	0.9
Repair rate (μ)	12/hr
Detection interval (T)	2
Failure rate of active unit (λ)	10–90 (varied)
Failure rate of standby unit (λs)	λ/4

The steady-state availability is calculated using the availability expression derived in Section 4. The active unit failure rate λ is varied between 10 and 90 to study its effect on system availability. Similarly, the repair rate μ is varied to assess its influence.

5.2 Availability vs Failure Rate

The availability of the system is calculated for different values of active unit failure rate. The results are shown as below.

Table 4. Availability vs. Failure Rate

Failure Rate of Active Unit (λ)	Availability
10	0.91
30	0.72
60	0.63
90	0.61

The relationship between failure rate and system availability is as shown in the figure below.

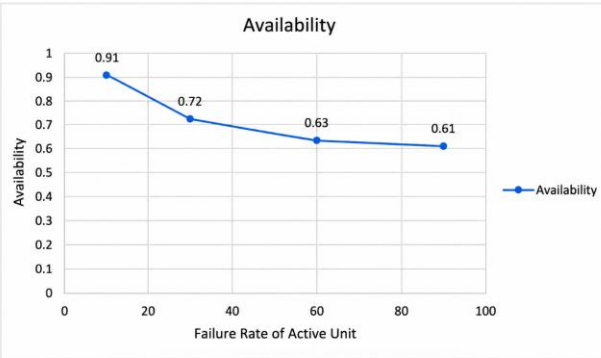

Fig 2: Failure Rate of Active Unit vs. Availability

The results presented in Table 4 and Fig 2 show that system availability decreases as the active unit failure rate increases. At a lower failure rate of 10, the system availability is high i.e. 0.91. As the failure rate moves to 30, 60 and 90, the availability keeps on decreasing to 0.72, 0.63 and 0.61. This indicates that component reliability has a direct influence on overall system performance and dependability.

5.3 Failure Rate on Availability

The result presented in Fig 2 clearly demonstrate that the system availability decreases as the failure rate of the active unit increases. With low failure rate, availability becomes high as the standby unit can take over the service when failure occur. With increase in the failure rate, it reduces the overall system availability.

It can be said that the system availability is highly dependent on the reliability of the active unit. So as the reliability improves it improve the overall system availability.

5.4 Availability vs Repair Rate

To study the effect of repair rate on system availability, the repair rate μ was varied while keeping the other system parameters constant. The parameter values used in this analysis are c=0.9, c_s=0.9, β=12, T=1, λ=30, and λs=λ/4. The system availability was calculated using the availability expression derived in section 4. Below are the availability values obtained for different repair rates.

Table 5. Availability vs. Repair Rate

Repair Rate (μ)	Availability
0.5	0.0777
1	0.1455
2	0.2578
4	0.4180
8	0.6006
12	0.6973

The relationship between repair rate and system availability is as shown in the figure below.

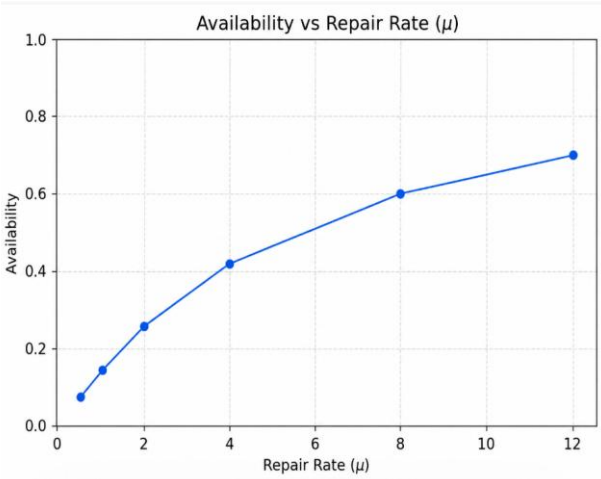

Fig 3: Repair rate vs Availability

It is observed that system availability increases as the repair rate increases. When the repair rate is low, failed units remain in the failed state for a longer time, which reduces system availability. As the repair rate increases, failed units are restored more quickly, which reduces system downtime and increases overall system availability.

Repair rate has a direct impact on system availability and dependability. When the maintenance and repair mechanism is improved the availability and dependability of active-passive cluster systems improve.

5.5 Dependability Discussion

Dependability of the system is closely related to system availability and reliability. Higher availability indicates higher dependability of the system. From the results, it can be observed that as the failure rate increases, the system dependability decreases. This shows that component reliability, repair rate, and failure detection mechanisms play an important role in maintaining system dependability.

The results obtained from the model can help system designers in selecting appropriate system parameters to improve availability and dependability of cluster systems.

From the parametric analysis, it is observed that improving the repair rate has a significant positive effect on system availability. The study suggests that system maintenance policies and repair efficiency are important factors in designing dependable cluster systems. With improved failure detection mechanism system downtime can be reduced and improve overall system availability and dependability.

5.6 Discussion of System Design Implications

The results obtained from the availability analysis show that both failure rate and repair rate have a significant effect on system availability and dependability. From the parametric analysis, it is observed that improving repair rate has a positive impact on system availability. System designers should focus on improving maintenance and repair mechanisms in order to reduce system downtime. Improving component reliability can significantly improve system availability and overall system dependability. The results of this study can be useful in designing high availability cluster systems where system downtime must be minimized.

5.7 Model Limitation

Different assumptions like exponential failure distribution and specific coverage probabilities are made while presenting the model. In real-world systems, system behavior may be different. Real system may follow more complex distribution in terms of failure behavior, repair processes, and failure detection mechanism. The model presented here is relatively simple. It contains single standby unit. It doesn't have multiple standby configurations or load sharing systems. Different external factor like preventive maintenance, human factors are not considered while building this model. With this all-mentioned point, the results presented in this paper should be interpreted with in the limitations of the assumptions used in the model.

6. CONCLUSION AND FUTURE WORK

In this paper, availability and dependability analysis of an active-passive cluster system was presented using a Semi-Markov model. Using the state diagram system behavior was modeled which includes active unit failure, standby unit failure, failure coverage detection mechanism. Steady-state balance equations are derived. Using the Semi-Markov approach the steady-state probabilities of system states are calculated. With the help of the calculated probabilities steady-state availability of the system is evaluated.

Performing the parametric analysis, we study the effect of active unit failure rate and repair rate on system availability. From the result we concluded that system availability decreases as the failure rate increases and increases as the repair rate increases. This indicates that the component reliability and repair efficiency play an important role in overall system availability and dependability. The result obtained can be used to design and evaluate high available cluster systems in distributed computing environments.

The work presented in this paper can be extended in several ways. Assumptions such as exponential failure distribution and specific coverage probability are done during the model presentation. The condition can be more complex in the real-time environment. As model can be complex in practical systems it can be extended further to include more realistic system behavior and additional system components. Future work may focus on extending the model to multi-node cluster systems and evaluating system availability under different maintenance and repair policies. The model can also be extended to include cost analysis and optimization of system parameters to achieve maximum availability. In addition, comparison between Markov and Semi-Markov models for different system configurations can be considered in future studies.

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