Controlled Target-Density Histogram Matching for Brightness-Preserving Contrast Enhancement in Medical Images

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ABSTRACT

This paper presents a brightness-preserving histogram matching framework for grayscale image enhancement with emphasis on medical imaging applications. A parametric center-shifted U-shaped target density is introduced to control intensity redistribution and reduce brightness distortion. Closed-form expressions for normalization and output mean are derived, enabling analytical control of enhancement behavior through minimization of the Absolute Mean Brightness Error (AMBE). Experimental results demonstrate that the proposed method achieves effective contrast enhancement with lower brightness distortion compared with Histogram Equalization (HE) and Contrast Limited Adaptive Histogram Equalization (CLAHE). An adaptive local extension (LU-EM) is also proposed to improve spatial adaptability and local detail enhancement. The results confirm that the proposed framework provides stable enhancement with effective brightness preservation for medical images.

General Terms

Image Processing; Medical Imaging; Digital Image Enhancement; Histogram-Based Enhancement; Mathematical Modeling.

Keywords

Contrast Enhancement; Histogram Matching; Histogram Equalization; CLAHE; Absolute Mean Brightness Error (AMBE); Medical Image Processing; Target Density; U-Shaped Distribution.

1. INTRODUCTION

Contrast enhancement is an important preprocessing step in digital image processing, particularly in medical imaging where low contrast may obscure diagnostically important structures. HE is widely used because of its simplicity and effectiveness; however, it often produces brightness distortion and over-enhancement artifacts. These limitations are especially problematic in medical images, where brightness consistency is important for reliable interpretation [6,7].

Several variants of histogram equalization have been developed to improve brightness preservation, including BBHE [1], DSIHE [2,5], MMBEBHE [3], and CLAHE [8]. Although these methods improve enhancement quality, most rely on heuristic histogram partitioning or clipping strategies and provide limited analytical control over the output distribution.

Histogram matching offers a more flexible framework by transforming an image toward a prescribed target distribution. The performance of this approach depends strongly on the design of the target probability density function. Motivated by this observation, this paper proposes a controlled target-density histogram matching framework based on a parametric center-shifted U-shaped distribution.

Closed-form expressions for normalization and output mean are derived, allowing analytical investigation of brightness behavior and parameter influence. Brightness preservation is evaluated using the AMBE. In addition, a local adaptive extension, referred to as the Local U-shaped Enhancement Method (LU-EM), is introduced to improve local contrast enhancement and spatial adaptability.

Experimental results on medical images demonstrate that the proposed methods achieve effective contrast enhancement while preserving brightness consistency and structural information. The remainder of this paper is organized as follows. Section 2 presents the continuous image model and histogram matching framework. Section 3 introduces the proposed U-shaped target density model. Section 4 presents experimental results for the global enhancement method, while Section 5 introduces the LU-EM framework and its experimental evaluation. Finally, conclusions are presented in Section 6.

2. CONTINUOUS IMAGE MODEL

Let $I(x,y)$ denote the gray-level intensity at pixel $(x,y)$of a grayscale image of size $M \times N$. For an 8-bit image, intensity values lie in the discrete set $\{0,1,\dots,L-1\}$ with $L = 256$.

Although the notation $I(x,y)$ reflects spatial dependence, the proposed framework operates on gray-level distributions. Therefore, the analysis focuses on the statistical properties of intensity values rather than their spatial coordinates.

The discrete histogram of the image is described by the probability mass function

$$p(r_k) = \frac{n_k}{MN}, k = 0,1,\dots,L-1, \qquad (1)$$

where n_k denotes the number of pixels having intensity r_k. The corresponding cumulative distribution function (CDF) is given by

$$T(r_k) = \sum_{i=0}^{k} p(r_i). \qquad (2)$$

For analytical convenience, a continuous formulation is adopted by normalizing gray levels to the interval $[0,1]$. In this setting, intensity values are modeled as realizations of a continuous random variable, where summations are replaced by integrals and the probability mass function is replaced by a probability density function (PDF).

Let A denote the normalized image, and let X represent the gray-level intensity of a randomly selected pixel. Denote by $f_X(x)$ and $F_X(x)$ the PDF and CDF of X, respectively. The objective is to construct a strictly increasing transformation g [9] such that

$$Y = g(X)$$

follows a prescribed target distribution with PDF $f_Y(y)$ and CDF $F_Y(y)$.

Since g is strictly increasing, the distribution of Y satisfies

$$F_Y(y) = F_X(g^{-1}(y)),$$

which leads to the fundamental histogram-matching identity

$$F_Y\big(g(x)\big) = F_X(x), \qquad x \in [0,1]. \qquad (3)$$

Accordingly, the transformation function is given by

$$g(x) = F_Y^{-1}(F_X(x)). \qquad (4)$$

Equation (4) forms the theoretical basis of histogram equalization and histogram specification [11]. Differentiating (3) with respect to x yields

$$f_Y(g(x))\, g'(x) = f_X(x),$$

or equivalently,

$$g'(x) = \frac{f_X(x)}{f_Y(g(x))}. \qquad (5)$$

Equation (5) provides important insight into the enhancement mechanism. The local behavior of the transformation is governed by the ratio between the input density and the target density. Regions where $g'(x) > 1$ undergo contrast expansion, while regions where $g'(x) < 1$ experience contrast compression.

For low-contrast images, the histogram is typically concentrated within a narrow intensity interval. In probabilistic terms, the density $f_X(x)$exhibits a dominant peak around a gray level $c \in (0,1)$, leading to reduced dynamic range and poor visual contrast.

To enhance contrast effectively, the transformation $g: [0,1] \to [0,1]$ should be continuous, strictly increasing, and differentiable. From (5), it follows that contrast enhancement becomes stronger when the denominator $f_Y(g(x))$ is relatively small. In particular, enhancement near the dominant intensity region around $x = c$ is promoted when

$$f_Y(g(c))$$

attains relatively small values.

This observation motivates the construction of target densities that are minimal near the dominant gray level and increase toward the intensity extremes. Such target distributions encourage intensity spreading away from the dominant region while preserving transformation stability. A natural and effective choice is therefore a U-shaped target density, which provides controlled contrast enhancement together with improved brightness preservation.

All medical images used in this study are obtained from the NIH Clinical Center Chest X-ray dataset.

3. CENTER-SHIFTED U-SHAPE MODEL

Unlike classical histogram equalization, which redistributes intensities uniformly, the proposed framework employs a controlled target distribution that emphasizes low and high gray levels while reducing the dominance of middle intensities. This design improves contrast near intensity extremes and avoids excessive concentration around mid-gray values.

To construct the enhanced image, the following parametric center-shifted U-shaped probability density function is introduced :

$$f_Y(y) = \alpha + \beta \mid y - c \mid^m, 0 \le y \le 1, \quad (11)$$

where $0 < c < 1$, $\alpha \ge 0$, $\beta \ge 0$, and $m > 0$.
The parameter c represents the center of the distribution and controls the location of minimum density. Consequently, the enhancement process can be adapted to the dominant gray-level region of the image. The parameter α determines the minimum value of the target density and therefore regulates the strength of contrast enhancement, while m controls the sharpness and curvature of the U-shaped profile.
Since $f_Y(y)$ is a probability density function, it must satisfy the normalization condition

$$\int_0^1 f_Y(y)\, dy = 1.$$

Substituting (11) into the normalization condition yields

$$\beta = \frac{(1-\alpha)(m+1)}{c^{m+1} + (1-c)^{m+1}}. \qquad (12)$$

Equation (12) ensures that the total probability mass remains equal to one for all admissible parameter values.
When $\beta = 0$, the distribution reduces to the uniform distribution associated with classical histogram equalization. Smaller values of α produce stronger U-shaped behavior and more aggressive enhancement, whereas larger values provide smoother intensity redistribution with improved brightness preservation. Increasing m increases the emphasis on dark and bright regions, while smaller values of m produce smoother enhancement behavior.
The proposed target density therefore provides analytical control over contrast enhancement and brightness preservation through a small number of interpretable parameters.

3.1 Brightness Preservation

Brightness preservation is evaluated using the AMBE:

$$\text{AMBE} = \mid \mu_Y - \mu_X \mid,$$

where μ_X and μ_Y denote the mean intensities of the input and enhanced images, respectively. Smaller AMBE values indicate better preservation of the original image brightness.

The mean intensity of the enhanced image is given by

$$\mu_Y = \int_0^1 y\, f_Y(y)\, dy.$$

Substituting the target density function defined in (11) yields

$$\mu_Y = \frac{\alpha}{2} + \beta\left(\frac{c^{m+2}}{(m+1)(m+2)} + \frac{(1-c)^{m+2}}{m+2} + \frac{c(1-c)^{m+1}}{m+1}\right).$$

The above expression provides a closed-form relationship between the output mean and the model parameters α, c, and

m. Consequently, the brightness behavior of the enhancement process can be analyzed analytically rather than empirically.

3.2 Case $m = 2$

For smooth and stable contrast enhancement, the quadratic case $m = 2$ is considered. In this case, the target density becomes sufficiently smooth while maintaining effective enhancement behavior. The output mean simplifies to

$$\mu_Y(\alpha, c) = \frac{\alpha}{2} + \beta\left(\frac{c^2}{2} - \frac{2c}{3} + \frac{1}{4}\right),$$

where

$$\beta = \frac{3(1-\alpha)}{c^3 + (1-c)^3}.$$

After simplification, the mean can be written in affine form as

$$\mu_Y(\alpha, c) = t(c) + \alpha\left(\frac{1}{2} - t(c)\right),$$

where

$$t(c) = \frac{6c^2 - 8c + 3}{4(3c^2 - 3c + 1)}.$$

This representation clearly illustrates the influence of the parameter α on brightness preservation. In particular, the output mean depends linearly on α for fixed c, which enables direct analytical adjustment of brightness behavior.

3.3 Optimization of Brightness Preservation

To minimize brightness distortion, the following objective function is introduced:

$$\phi(c, \alpha) = |\, \mu_Y(c, \alpha) - \mu_X \,|. \qquad (13)$$

The optimization is performed over the feasible parameter region

$$R = \{(c, \alpha): 0 \le c \le 1,\ 0 \le \alpha \le 1\}.$$

Since $\phi$is continuous on the closed and bounded set R, the Extreme Value Theorem guarantees the existence of at least one minimizer $(c^*, \alpha^*) \in R$such that

$$\mu_Y(c^*, \alpha^*) \approx \mu_X.$$

Therefore, the proposed framework provides a mathematically well-defined mechanism for brightness-preserving enhancement.

For stationary analysis, the absolute value is temporarily ignored. Differentiating with respect to $c$gives

$$\frac{\partial \phi}{\partial c} = 0 \Rightarrow t'(c)(1-\alpha) = 0.$$

Except for the trivial uniform-distribution case $\alpha = 1$, the stationary condition reduces to

$$t'(c) = 0.$$

Solving this equation yields

$$c = \frac{1}{2} \pm \frac{\sqrt{3}}{6}.$$

Similarly, differentiating with respect to α gives

$$\frac{\partial \phi}{\partial \alpha} = 0 \Rightarrow t(c) = \frac{1}{2},$$

which implies

$$c = \frac{1}{2}.$$

Consequently, the stationary analysis leads to the following candidate optimal center values :

$$c^* \in \left\{\frac{1}{2},\ \frac{1}{2} \pm \frac{\sqrt{3}}{6}\right\}.$$

These values determine suitable positions for the minimum of the target density and therefore provide effective analytical control over brightness preservation.

For dark images, the dominant histogram mass is concentrated near lower gray levels. In this case, the optimal center parameter is selected as

$$c = \frac{1}{2} + \frac{\sqrt{3}}{6},$$

which shifts the minimum of the target density toward higher intensities and promotes expansion of dark regions. Conversely, for bright images, the dominant histogram mass is concentrated near higher gray levels, and the parameter

$$c = \frac{1}{2} - \frac{\sqrt{3}}{6}$$

provides more balanced intensity redistribution and improved brightness preservation. Consequently, the sign of the optimal shift is determined according to the dominant brightness characteristics of the input image.

3.4 Optimal Choice of α for Fixed c

For a fixed center parameter c, the value of α that preserves the input mean can be obtained directly by solving

$$\mu_Y(\alpha, c) = \mu_X.$$

This yields

$$\alpha^*(c) = \frac{\mu_X - t(c)}{\frac{1}{2} - t(c)}, t(c) \neq \frac{1}{2}.$$

The above expression provides a direct analytical formula for selecting the enhancement parameter α based on the mean intensity of the input image. Consequently, the proposed framework allows explicit control of brightness preservation without requiring iterative optimization procedures.

4. EXPERIMENTAL RESULT WITH OPTIMAL PARAMETERS

4.1 Optimal Parameter Configuration

Using the analytical optimization framework developed in the previous section, the optimal parameter values were obtained as

$$c^* = \frac{1}{2} + \frac{\sqrt{3}}{6}, \alpha^* = 0.7040.$$

These parameter values minimize the brightness distortion measured by the AMBE while maintaining effective contrast enhancement.

Figure 1 presents a visual comparison between the original image, HE, and the proposed U-shaped Enhancement Method (U-EM). The HE result increases global contrast but introduces noticeable brightness modification and over-enhancement in

several image regions. In contrast, the proposed method enhances structural and anatomical details while preserving the overall brightness characteristics of the original image. The enhanced image produced by the proposed framework appears visually more balanced and maintains greater consistency with the original intensity distribution.

Table 1 summarizes the quantitative performance evaluation using Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index Measure (SSIM), entropy, and Absolute Mean Brightness Error (AMBE) [10,11]. The experimental results indicate that the proposed method achieves superior brightness preservation compared with classical histogram equalization. In particular, HE produces a relatively large brightness distortion with

$$\text{AMBE} = 0.0890,$$

whereas the proposed method reduces the brightness error to approximately

$$\text{AMBE} \approx 0.0005.$$

In addition, the proposed framework achieves improved image fidelity and structural preservation, obtaining a PSNR value of 25.097 dBand an SSIM value of 0.9043. These results indicate that the enhanced image remains structurally closer to the original image while still achieving effective contrast enhancement.

The entropy values further confirm that the proposed framework increases the distribution of useful intensity information without introducing excessive enhancement artifacts. Unlike HE, which redistributes intensities uniformly and may amplify noise or distort image appearance, the proposed U-shaped target density provides controlled intensity redistribution through analytically adjustable parameters.

Table 1: Quantitative comparison of the proposed U-shaped enhancement method (U-EM)

Method	PSNR (dB)	SSIM	AMBE	Entropy
Original	∞	1.0000	0.0000	7.5976
HE	20.464	0.8409	0.0890	7.3821
U-EM	25.160	0.9041	0.0005	7.3358

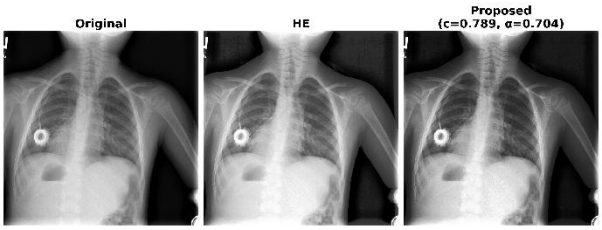

Fig 1: Original image, HE, and the proposed U-EM with $\mathbf{c = 0.7887}$, $\mathbf{m = 2}$, and $\boldsymbol{\alpha = 0.7040}$.

4.2 Bright Image Case

The proposed framework was further evaluated using a bright medical image characterized by a narrow dynamic range and concentration of intensities within the higher gray-level region. Such images typically present limited visual contrast and reduced visibility of subtle anatomical structures.

Table 2 presents the quantitative comparison between the original image, HE, and the proposed U-shaped Enhancement Method (U-EM). The proposed method was implemented using the parameter values

$$c = 0.2113, \qquad m = 2, \qquad \alpha = 0.5498.$$

Figure 2 shows the corresponding visual enhancement results. Classical histogram equalization increases global contrast but introduces substantial brightness distortion and noticeable changes in overall image appearance. In contrast, the proposed method enhances structural details while maintaining brightness consistency with the original image.

The quantitative results demonstrate that HE produces significant brightness distortion with

$$\text{AMBE} = 0.1262,$$

whereas the proposed U-EM reduces the brightness error to

$$\text{AMBE} = 0.0019.$$

At the same time, the proposed framework achieves improved structural preservation with an SSIM value of 0.8902, compared with $0.7942$for HE. The PSNR value is also improved, increasing from 14.828 dBfor HE to 18.864 dBfor the proposed method. Although the entropy values of both enhancement methods remain comparable, the proposed framework achieves more stable intensity redistribution without introducing excessive brightness variation.

These results confirm that selecting the center parameter c according to the statistical characteristics of the image enables effective analytical control of brightness preservation. The proposed method consistently achieves substantially lower AMBE values while maintaining competitive PSNR and SSIM performance. This behavior demonstrates that the proposed framework provides effective contrast enhancement together with minimal brightness distortion, which is particularly important in medical imaging applications where brightness consistency is essential for reliable visual interpretation and diagnostic analysis.

Table 2: Quantitative comparison for the bright image case

Method	PSNR (dB)	SSIM	AMBE	Entropy
Original	∞	1.0000	0.0000	7.3208
HE	14.828	0.7942	0.1262	7.1434
U-EM	18.864	0.8902	0.0019	7.1305

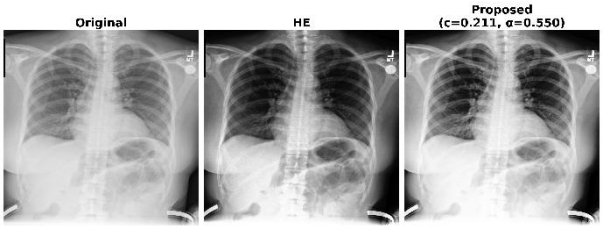

Fig 2: Original image, histogram equalization (HE), and the proposed U-EM with $\mathbf{c = 0.2113}$, $\mathbf{m = 2}$, and $\boldsymbol{\alpha = 0.5498}$.

5. ADAPTIVE LOCAL U-SHAPED ENHANCEMENT (LU-EM)

5.1 Algorithm for Local U-Enhancement Method

Although the global U-shaped enhancement framework provides effective brightness-preserving contrast enhancement,

medical images frequently contain spatially varying illumination and locally distributed anatomical structures. In such cases, a global transformation may not sufficiently enhance fine local details in all image regions. To address this limitation, the proposed framework is extended to a locally adaptive enhancement scheme referred to as the Local U-shaped Enhancement Method (LU-EM).

The proposed LU-EM is conceptually related to Contrast CLAHE, since the image is partitioned into smaller regions and enhancement is performed locally. However, unlike CLAHE, which relies on clipped histogram equalization, the proposed approach employs analytically defined U-shaped target distributions with adaptive local parameters. This provides explicit probabilistic control over local intensity redistribution while maintaining brightness consistency.

Let the normalized gray level be $r \in [0,1]$. The image is divided into non-overlapping tiles Ω_k, and the following procedure is applied to each tile.

Step 1: Local Histogram Estimation
For each tile Ω_k, the local histogram is computed and represented by the probability density function

$$p_k(r).$$

This density describes the local gray-level distribution within the tile.

Step 2: Local Cumulative Distribution Function
The corresponding local cumulative distribution function is computed as

$$F_k(r) = \int_0^r p_k(t)\, dt.$$

The function $F_k$characterizes the cumulative local intensity behavior of the tile.

Step 3: Construction of the Local Target Distribution
A locally adaptive U-shaped target density $q(r)$is then defined with center parameter c. The corresponding cumulative distribution function is given by

$$G(r) = \int_0^r q(t)\, dt.$$

The U-shaped target density promotes intensity spreading away from dominant local gray levels, thereby improving local contrast while preserving structural stability.

Step 4: Local Histogram Matching
Histogram matching is subsequently performed using the transformation

$$T_k(r) = G^{-1}(F_k(r)).$$

This transformation maps the local histogram of each tile to the prescribed U-shaped target distribution.
The enhancement process is applied independently to all image tiles. Since independent local processing may generate discontinuities at tile boundaries, bilinear interpolation is employed to smoothly combine neighboring regions, similarly to CLAHE. This interpolation step reduces blocking artifacts and produces spatially continuous enhancement results.

To introduce spatial adaptivity while maintaining smooth transitions, the center parameter of the U-shaped target density is selected according to the local brightness characteristics of the image. Specifically, the center parameter is defined by

$$c = 1 - \tilde{\mu}_k,$$

where $\tilde{\mu}_k$ denotes a smoothed local mean intensity. Consequently, darker local regions produce larger values of c, whereas brighter regions generate smaller values. This adaptive formulation enables the target density to respond dynamically to local brightness variations while avoiding abrupt parameter changes between neighboring tiles.

In addition, Gaussian-weighted aggregation together with a light post-smoothing operation is employed to further improve spatial consistency and suppress enhancement artifacts. These steps stabilize the enhancement process and reduce excessive local intensity fluctuations, particularly in homogeneous image regions.
The proposed LU-EM framework therefore combines the advantages of local adaptive enhancement with the analytical control provided by the proposed U-shaped target density model. As a result, the method enhances fine local structures while maintaining brightness preservation and spatial smoothness, which are particularly important in medical imaging applications.

5.2 Experimental Results

Tables 3–5 present quantitative comparisons between the proposed Local U-shaped Enhancement Method (LU-EM) and CLAHE. The results show that LU-EM consistently achieves higher PSNR values while maintaining competitive or improved SSIM and better brightness preservation.
Unlike CLAHE, which may produce excessive local contrast enhancement and amplify noise, the proposed LU-EM employs analytically controlled U-shaped target distributions with adaptive local parameters. This enables more stable intensity redistribution and smoother enhancement behavior.
In Table 3, LU-EM improves PSNR from 23.7082 dBto 26.6574 dBand increases SSIM from 0.8858 to 0.92086. In Table 4, the proposed method achieves substantial improvement in both PSNR and AMBE, reducing brightness distortion from 0.0270 to 0.0045. Similar behavior is observed in Table 5, where LU-EM provides improved structural preservation and enhanced entropy characteristics.

Figures 3–5 visually demonstrate that LU-EM enhances anatomical structures while maintaining smoother brightness transitions and reducing over-enhancement artifacts. The adaptive center parameter

$$c_k = 1 - \tilde{\mu}_k$$

allows the enhancement process to adapt to local brightness characteristics, thereby improving spatial consistency and brightness preservation.

Overall, the proposed LU-EM framework provides an effective balance between local contrast enhancement, structural preservation, and brightness consistency, making it suitable for medical imaging applications.

Table 3: Quantitative comparison of the LU-EM **method**

Method	PSNR (dB)	SSIM	AMBE	Entropy
Original	∞	1.0000	0.0000	7.3208
CLAHE	23.7082	0.8858	0.0005	7.5016
LU-EM	26.6574	0.92086	0.01203	7.4663

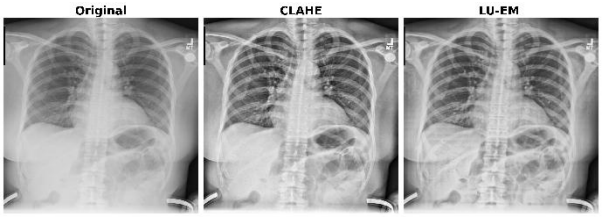

Fig 3: The original image, CLAHE, and the LU-EM with $c = 1 - \tilde{\mu}$, m=2, and for α =0.5

Table 4: Quantitative comparison of the LU-EM method

Method	PSNR (dB)	SSIM	AMBE	Entropy
Original	∞	1.0000	0.0000	7.7165
CLAHE	21.9921	0.8519	0.0270	7.7687
LU-EM	27.3984	0.8701	0.0045	7.8363

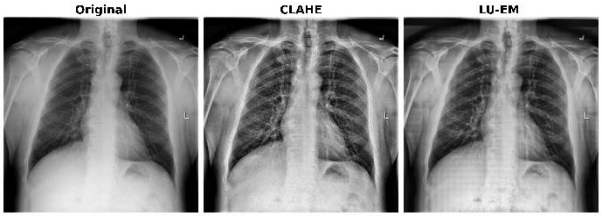

Fig 4: The original image, CLAHE, and the proposed LU-EM with $c = 1 - \tilde{\mu}$, m=2, and for α =0.5

Table 5: Quantitative comparison of the LU-EM method

Method	PSNR (dB)	SSIM	AMBE	Entropy
Original	∞	1.0000	0.0000	7.6632
CLAHE	21.3544	0.8448	0.0282	7.7890
LU-EM	24.7919	0.8537	0.01765	7.8180

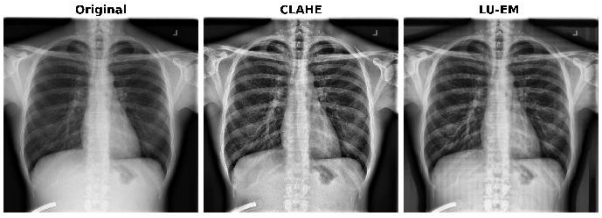

Fig 5: The original image, CLAHE, and the LU-EM with $c = 1 - \tilde{\mu}_k$, m=2, and for α =0.5

6. CONCLUSION

This paper introduced a U-shaped target-density histogram matching framework for grayscale image enhancement. The proposed method provides analytical control of intensity redistribution and brightness preservation through adjustable model parameters. A local adaptive extension (LU-EM) was also developed to improve spatial adaptability and local detail enhancement.

Experimental results on medical images demonstrated that the proposed methods effectively enhance contrast while preserving brightness consistency and structural information. The global model achieved significantly lower brightness distortion than classical histogram equalization, while LU-EM produced higher PSNR values and competitive SSIM performance compared with CLAHE.

Overall, the proposed framework provides a stable and effective approach for brightness-preserving contrast enhancement in medical imaging applications. Future work will consider adaptive parameter selection, color image enhancement, and extensions to multimodal medical imaging.

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