Stepwise Image Contrast Enhancement via Variance-Bounded Intensity Scaling

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ABSTRACT
Low-contrast images frequently arise due to poor illumination, limited sensor dynamic range, or unfavorable acquisition conditions, leading to compressed gray-level distributions and degraded visual quality. Although Histogram Equalization (HE) is widely used for contrast enhancement, it often results in over-enhancement, brightness distortion, and noise amplification, particularly in low-contrast scenarios.

This paper presents a stepwise, threshold-centered intensity transformation for enhancing low-contrast images within a continuous gray-level framework, normalized to the interval $[0,1]$. The proposed method employs a threshold Tdetermined by the mean intensity of the input image, enabling adaptive localization of the dominant intensity level. The image variance is utilized to define the width of enhancement intervals around the threshold, while gain parameters regulate the amplification strength in a controlled and bounded manner.

Experimental results demonstrate that the proposed method enhances contrast and preserves structural details while maintaining brightness consistency. Quantitative evaluations indicate superior performance compared to classical histogram equalization in terms of PSNR, SSIM, and AMBE, confirming the effectiveness and robustness of the proposed approach.

General Terms
Image Processing, Algorithms, Pattern Recognition.

Keywords
Contrast enhancement; stepwise intensity transformation; variance-bounded mapping; low-contrast image enhancement; brightness preservation

1. INTRODUCTION
Contrast enhancement is a fundamental task in image processing, as many images acquired under non-ideal illumination conditions exhibit low contrast and degraded visual quality. Classical enhancement techniques, most notably Histogram Equalization (HE) [1], aim to improve contrast by redistributing gray-level intensities over the available dynamic range. However, such methods often lead to over-enhancement, noise amplification, and loss of structural information. In particular, several studies have highlighted the limitations of standard HE with respect to brightness distortion and excessive contrast stretching [4,5]. Furthermore, it has been recognized that effective contrast enhancement should be properly bounded in order to preserve image fidelity and visual stability [7,8].

To address these limitations, various adaptive intensity transformation methods have been proposed. Nevertheless, many existing approaches rely on global image statistics or highly nonlinear mappings, which may lack stability guarantees and can introduce undesirable visual artifacts. These challenges have motivated the development of piecewise intensity transformation techniques, which provide greater control over the enhancement process and allow for more interpretable behavior [6].

This paper presents a novel stepwise intensity transformation, denoted by F_λ, for contrast enhancement. The proposed method is based on variance-bounded multiplicative scaling centered at an adaptive threshold determined by the mean intensity of the input image. Specifically, the intensity domain is partitioned into bounded step intervals, within which controlled amplification or attenuation is applied. An explicit upper bound on the number of steps is derived to ensure that the transformation remains within the admissible intensity range. This formulation yields a globally non-decreasing mapping with stable enhancement behavior, while avoiding excessive clipping and distortion.

Experimental results demonstrate that the proposed method consistently outperforms classical histogram equalization in terms of PSNR, SSIM [3], and entropy [9]. In addition, the method achieves improved brightness preservation and produces visually natural images with enhanced contrast and strong structural fidelity.

2. ONE-STEP λ-BASED TRANSFORMATION
To motivate the proposed framework, the effect of a one-step intensity transformation is first examined, highlighting the need for a more stable multi-step enhancement strategy; see, for example, [10].

Let $I(x,y) \in [0,1]$ denote the normalized gray-level intensity of an image. Let T be a threshold determined by the mean intensity of I, and let β denote the global variance of I. Furthermore, let $\lambda_1, \lambda_2 > 0$ be control parameters.

The one-step λ-based transformation F_λ is defined as the composition

$$F_\lambda = F_2 \circ F_1,$$

where the first mapping $F_1$is given by

$$F_1(x,y) = \begin{cases} (1+\lambda_1\beta)\, I(x,y), & I(x,y) > T, \\ (1-\lambda_2\beta)\, I(x,y), & I(x,y) \le T, \end{cases}$$

and the second mapping $F_2$is defined as

$$F_2(\tau) = \begin{cases} \dfrac{1}{1+\lambda_1\beta}\,\tau, & \tau > 1, \\ \tau, & \tau \le 1. \end{cases}$$

The mapping F_1 amplifies intensities above the threshold T and attenuates those below or equal to T, with the enhancement strength governed by the parameters λ_1 and λ_2, scaled by the variance β. However, this amplification may produce values exceeding the admissible range [0,1]. The second mapping F_2 is therefore introduced as a corrective step that rescales only those values exceeding unity, ensuring that all output intensities remain within the valid range.

3. STEPWISE INTENSITY TRANSFORMATION

Motivated by the limitations observed in Section 2, we introduce a multi-step, piecewise continuous intensity transformation centered at the threshold T that enables adaptive contrast enhancement using multiple statistically driven factors

3.1 Definition of the Stepwise Transformation

Let $\lambda_1, \lambda_2 > 0$, $0 \le \beta \le \frac{1}{2}$, and $0 \le T \le 1$. Define the characteristic function

$$\phi_{[a,b]}: \mathbb{R} \to \{0,1\} \text{ by } \phi_{[a,b]}(x) = \begin{cases} 1, & x \in [a,b], \\ 0, & \text{otherwise.} \end{cases}$$

Three auxiliary functions

$F_{\lambda_1}, F_I, F_{\lambda_2}: [0,1] \to \mathbb{R}$ are defined as follows:

$$F_{\lambda_1}(\tau) = \sum_{k=1}^{n} (1 + k\,\lambda_1\beta)\,\tau\,\phi_{[(k-1)\beta,\,k\beta]}(\tau - T),$$

$$F_I(\tau) = \tau\,\phi_{[-\beta,\,0]}(\tau - T),$$

$$F_{\lambda_2}(\tau) = \sum_{k=1}^{n} (1 - k\,\lambda_2\beta)\,\tau\,\phi_{[-(k+1)\beta,\,-k\beta]}(\tau - T).$$

The proposed stepwise transformation is then defined by

$$F(\tau) = F_{\lambda_1}(\tau) + F_I(\tau) + F_{\lambda_2}(\tau).$$

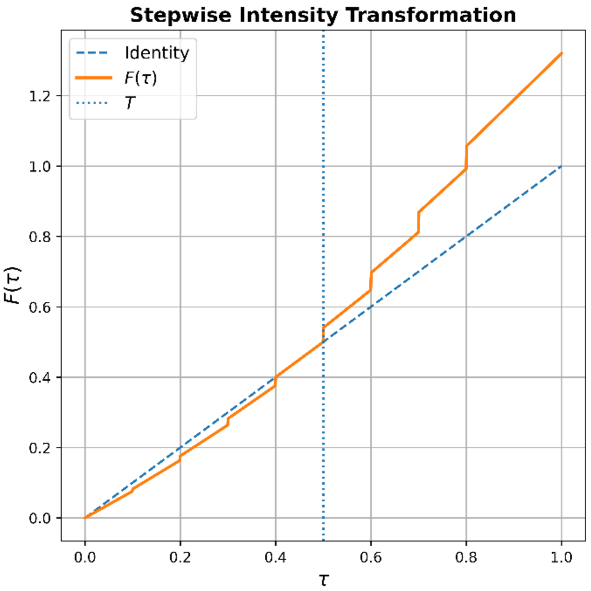

Fig 1: illustrates the stepwise nature of the proposed transformation, where the slope increases above the threshold and decreases below it

3.2 Final Piecewise Form

The proposed transformation defines a monotone, multi-step, piecewise-linear intensity mapping centered at the threshold T. The stepwise modulation of slopes across intensity intervals enables controlled contrast amplification above the threshold and attenuation below it, governed by the parameters λ_1 and λ_2.

By construction, the multiplicative gains increase monotonically for intensities above T and decrease monotonically for intensities below T. This structure ensures that the transformation remains globally non-decreasing under appropriate parameter constraints.

The transformation admits the following explicit piecewise form:

$$F(\tau) = \begin{cases} (1 + n\lambda_1\beta)\,\tau, & n\beta < \tau - T \le (n+1)\beta, \\ \vdots & \\ (1 + k\lambda_1\beta)\,\tau, & (k-1)\beta < \tau - T \le k\beta, \\ \vdots & \\ (1 + \lambda_1\beta)\,\tau, & 0 < \tau - T \le \beta, \\ \tau, & -\beta \le \tau - T \le 0, \\ (1 - \lambda_2\beta)\,\tau, & -2\beta \le \tau - T < -\beta, \\ \vdots & \\ (1 - k\lambda_2\beta)\,\tau, & -(k+1)\beta \le \tau - T < -k\beta, \\ \vdots & \\ (1 - n\lambda_2\beta)\,\tau, & -(n+1)\beta \le \tau - T < -n\beta. \end{cases}$$

To guarantee monotonicity, it is sufficient to impose the condition

$$1 - n\lambda_2\beta \ge 0,$$

which ensures that all multiplicative factors remain non-negative. Under this condition, the mapping F is globally non-decreasing on [0,1].

3.3 Upper Bound on the Number of Steps and Adaptive Parameter Selection

Let $T \in (0,1)$, $\beta = \mathrm{Var}\,(I) > 0$, and $\lambda_1, \lambda_2 > 0$

3.3.1 Upper Intensity Constraint

The maximum value of the transformation occurs at the highest positive step, given by

$$F_{\max} = (1 + n\lambda_1\beta)(T + n\beta).$$

To ensure that the transformation remains within the admissible range [0,1], the condition

$$(1 + n\lambda_1\beta)(T + n\beta) \le 1$$

must be satisfied.

Expanding this expression yields the quadratic inequality

$$\lambda_1\beta^2 n^2 + (\beta + \lambda_1\beta T)n + (T - 1) \le 0.$$

The corresponding discriminant is

$$\Delta = (\beta + \lambda_1\beta T)^2 - 4\lambda_1\beta^2(T - 1)$$
$$= \beta^2\big((1 + \lambda_1 T)^2 + 4\lambda_1(1 - T)\big) > 0.$$

Thus, the admissible values of n satisfy

$$0 \le n \le \frac{-(1 + \lambda_1 T) + \sqrt{(1 + \lambda_1 T)^2 + 4\lambda_1(1 - T)}}{2\lambda_1\beta}. \quad (1)$$

3.3.2 Lower Intensity Constraint

The minimum value of the transformation occurs at the lowest negative step:

$$F_{\min} = (1 - n\lambda_2\beta)(T - (n+1)\beta).$$

To guarantee non-negativity, we require $F_{\min} \ge 0$, which leads to the conditions

$$n \leq \frac{1}{\lambda_2 \beta} \ \ and \ \ n \leq \frac{T}{\beta} - 1. \qquad (2)$$

3.3.3 Final Upper Bound

Combining (1) and (2), the number of admissible steps is bounded by

$$n \leq n_{\max} = \min\left(\frac{T}{\beta} - 1,\ \frac{1}{\lambda_2 \beta},\ \frac{-(1+\lambda_1 T) + \sqrt{(1+\lambda_1 T)^2 + 4\lambda_1 (1-T)}}{2\lambda_1 \beta}\right).$$

This bound ensures that the transformation remains within the valid intensity range while preserving monotonicity.

3.3.4 Adaptive Selection of $\boldsymbol{T}$, $\boldsymbol{\lambda_1}$, and $\boldsymbol{\lambda_2}$

To adapt the transformation to the statistical characteristics of the input image, the parameters λ_1 and λ_2 are selected based on the mean intensity

$$\mu = \frac{1}{MN} \sum_{x,y} I(x, y).$$

The parameters are chosen according to the rule

$$\lambda_1 = \begin{cases} [0.6, 1.0], & \mu < 0.5, \\ [0.3, 0.6], & \mu \geq 0.5, \end{cases} \quad \lambda_2 = \begin{cases} [0.3, 0.6], & \mu < 0.5, \\ [0.6, 1.0], & \mu \geq 0.5. \end{cases}$$

For dark images ($\mu < 0.5$), larger values of λ_1 enhance intensities above the threshold T, while smaller values of λ_2 prevent excessive suppression of darker regions. Conversely, for bright images ($\mu \geq 0.5$), larger values of λ_2 attenuate high intensities, while smaller values of λ_1 avoid over-enhancement.

Since the mean intensity reflects the global brightness level of the image, the threshold T is naturally chosen as

$$T = \mu.$$

This adaptive parameter selection enables the proposed transformation to balance contrast enhancement and brightness preservation in a data-driven manner.

4. VISUAL COMPARISON

The following figures present visual comparisons between the original images, histogram equalization (HE), and the proposed method. All test images are taken from the Kodak dataset. The experimental evaluation is supported by both qualitative results (Figures 1–5) and quantitative analysis presented in Section 5.

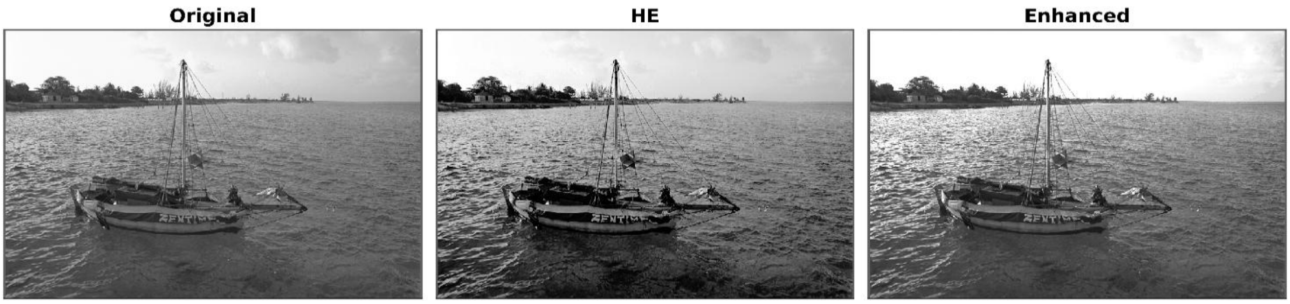

Fig. 1. Visual comparison of the original image, histogram equalization (HE), and the proposed enhanced result

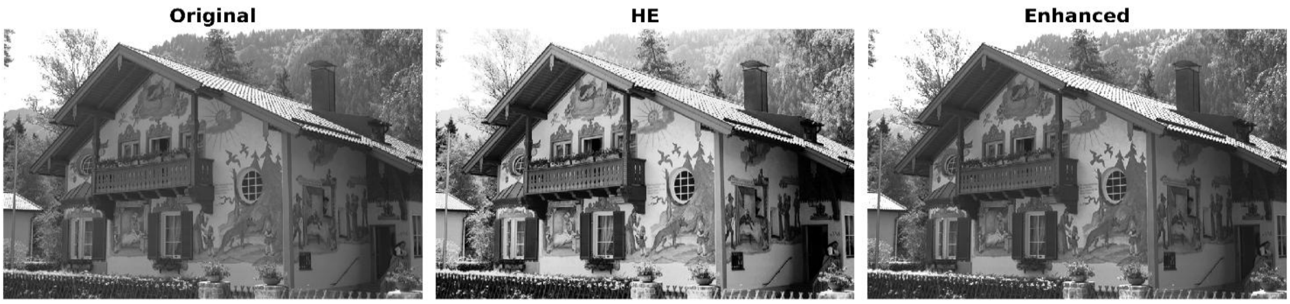

Fig. 2. Visual comparison of the original image, histogram equalization (HE), and the proposed enhanced result

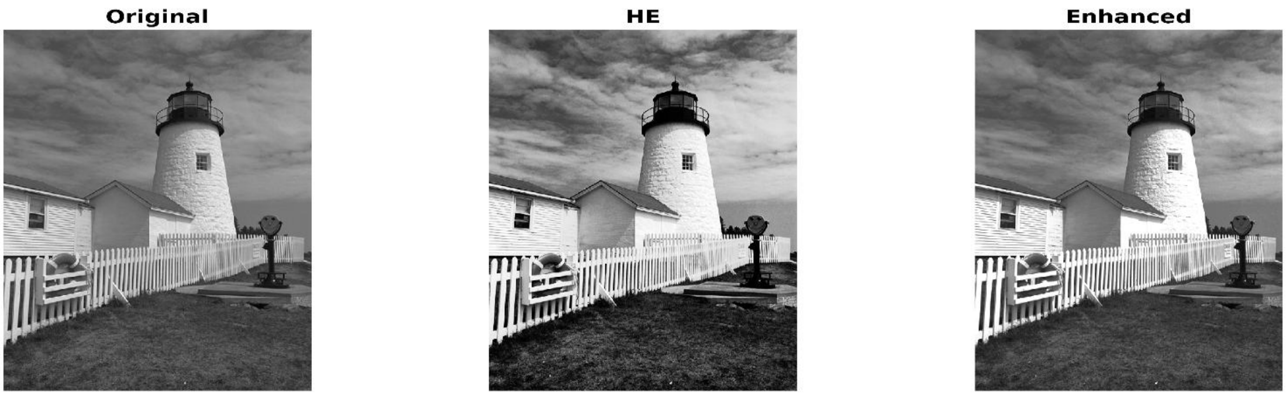

Fig. 3. Visual comparison of the original image, histogram equalization (HE), and the proposed enhanced result.

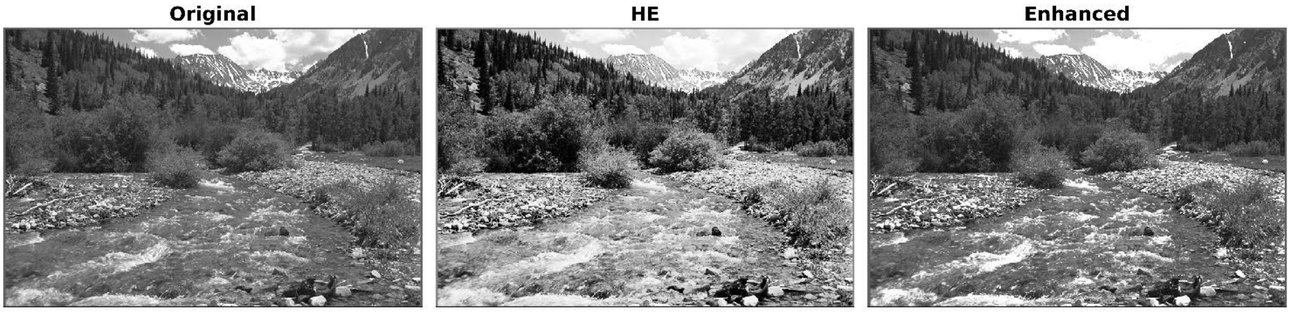

Fig. 4. Visual comparison of the original image, histogram equalization (HE), and the proposed enhanced result.

Fig. 5. Visual comparison of the original image, histogram equalization (HE), and the proposed enhanced result.

The proposed method produces visually balanced images with enhanced contrast while avoiding over-enhancement artifacts observed in histogram equalization.

5. QUANTITATIVE COMPARISON AND EXPERIMENTAL RESULTS.

To evaluate the effectiveness of the proposed stepwise transformation, a series of experiments were conducted on multiple grayscale images. The performance of the proposed method was compared with classical histogram equalization (HE) using standard quantitative metrics, including Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index (SSIM), entropy, and Absolute Mean Brightness Error (AMBE).

All entropy values were computed using a 256-bin histogram (base-2), consistent with MATLAB's entropy definition.

Table 1. Quantitative comparison and parameter settings (Image 1)

Metric/Parameter	Original	HE	Proposed
PSNR (dB)	∞	21.73	**24.89**
SSIM	1.0000	0.8611	**0.9606**
Entropy	7.4579	7.2240	6.7269
AMBE	0.0000	0.0219	**0.0004**
Mean	0.5256	0.5037	0.5251
Variance	0.0556	0.0842	0.0815
Parameters	—	—	$\lambda_1 = 0.5$, $\lambda_2 = 1.0$ T =0.5256, n = 5

Table 2. Quantitative comparison and parameter settings (Image 2)

Metric / Parameter	Original	HE	Proposed
PSNR (dB)	∞	16.45	**25.10**
SSIM	1.0000	0.7735	**0.9547**
Entropy	7.1146	6.8902	6.7893
AMBE	0.0000	0.0772	**0.0176**
Mean	0.4275	0.5047	0.4451
Variance	0.0441	0.0841	0.0592
Parameters	—	—	$\lambda_1 = 1.0$, $\lambda_2 = 0.5$, T = 0.4275, n = 7

Table 3. Quantitative comparison and parameter settings (Image 3)

Metric / Parameter	Original	HE	Proposed
PSNR (dB)	∞	17.53	**22.57**
SSIM	1.0000	0.8244	**0.9495**
Entropy	7.3786	7.1328	6.8029
AMBE	0.0000	0.0496	**0.0322**
Mean	0.4535	0.5031	0.4857
Variance	0.0337	0.0836	0.0598
Parameters	—	—	$\lambda_1 = 1.0$, $\lambda_2 = 0.5$ T = 0.4500, n = 9

Table 4. Quantitative comparison and parameter settings (Image 4)

Metric/Parameter	Original	HE	Proposed
PSNR (dB)	∞	16.83	**23.10**
SSIM	1.0000	0.7925	**0.9423**
Entropy	7.4328	7.1752	6.8603
AMBE	0.0000	0.0799	**0.0265**
Mean	0.4228	0.5027	0.4494
Variance	0.0429	0.0835	0.0665
Parameters	—	—	$\lambda_1 = 1.0$, $\lambda_2 = 0.5$, T = 0.4228, n = 7

Table 5. Quantitative comparison and parameter settings (Image 5)

Metric / Parameter	Original	HE	Proposed
PSNR (dB)	∞	16.22	**23.57**
SSIM	1.0000	0.7350	**0.9535**
Entropy	7.1264	6.8721	6.9100
AMBE	0.0000	0.0286	**0.0277**
Mean	0.4758	0.5044	0.5035
Variance	0.0239	0.0834	0.0424
Parameters	—	—	$\lambda_1 = 1.0$, $\lambda_2 = 0.5$ T = 0.4758, n = 12

The results consistently demonstrate that the proposed method achieves higher PSNR and SSIM compared to histogram equalization, while significantly reducing AMBE. This indicates improved contrast enhancement with better brightness preservation and structural fidelity.

These results confirm the robustness and stability of the proposed method across different image characteristics and contrast conditions.

6. CONCLUSION

This paper presents a novel stepwise contrast enhancement method based on variance-bounded intensity scaling. The proposed approach introduces a threshold-centered, multi-step transformation that adaptively modifies image intensities using variance-controlled multiplicative gains. In contrast to classical histogram equalization, which often leads to over-enhancement and brightness distortion, the proposed method provides a controlled and stable enhancement mechanism through a bounded stepwise mapping.

The transformation is formulated within a normalized gray-level framework and incorporates an explicit upper bound on the number of steps to ensure that output intensities remain within the admissible range [0,1]. The use of image variance to define the width of step intervals, together with an adaptive threshold based on the mean intensity, enables the method to adjust naturally to different image characteristics. Furthermore, the proposed parameter selection strategy for λ_1 and λ_2 ensures balanced enhancement behavior for both dark and bright images.

Experimental results demonstrate that the proposed method consistently outperforms classical histogram equalization in terms of PSNR and SSIM, while significantly reducing AMBE. These results indicate that the method effectively enhances contrast while preserving brightness and structural details, producing visually natural results without introducing artifacts or excessive amplification.

Overall, the proposed framework provides a simple yet effective alternative to traditional contrast enhancement techniques, combining theoretical guarantees with practical adaptability.

Future work will focus on extending the proposed framework to locally adaptive enhancement schemes, where transformation parameters are computed for image tiles to further improve performance under non-uniform illumination conditions. In addition, smoothing strategies may be investigated to reduce possible discontinuities between adjacent steps while preserving the advantages of the stepwise structure.

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