An Approach to Risk Analysis for Construction Project using Fuzzy Logic

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ABSTRACT
Project managers play very important role in the execution of the complete project whether it would be the initiation stage, planning stage, execution stage, monitoring and controlling or the closing stages of the project under the restriction of a variety of constraints. Therefore, it is extremely important that these constraints can be controlled and managed simultaneously. Fuzzy analysis approach explains risk reasoning process in the field of risk analysis and management. In this paper we will present an effective technique to model risk analysis with construction project using fuzzy triangular representation and its adaptation on membership function which expresses the project productivity graphically. The model proposed in this paper assesses the related risks with a construction operation based on linguistic approximation of evaluations made by three experts or associated professionals. The outcomes got from this model demonstrate a precise and compelling way for risk investigation of any project related to many activities.

Keywords
Fuzzy Risk, Fuzzy Membership function, Fuzzy Weighted Average (FWA), Linguistic Approximation, Natural Language representation

1. INTRODUCTION
"A systematic way of looking at areas of risk and consciously determining how each should be treated which aims at identifying sources of risk and uncertainty, determining their impact, and developing appropriate management response is called Risk Management" (Uher, 2003). Methods for risk investigation can be either subjective or quantitative relying upon the data accessible and the degree of detail that is required (Bennett J.C. Bohoris G.A.,Aspinwall E.M.,Hall R.C. et. al 1996). Quantitative methodology is the primary thought for quantitative procedures. Subjective methods depend more on judgment than on factual estimations like Scenario investigation (R.K.J.R. Rainer, et al. 1991), Fuzzy Set theory (R.K.J.R. Rainer, et al. 1991) and so forth. Quantitative methods can include huge extra costs and is just justified in the uncommon occurrence where the presumptions of likelihood hypothesis apply. Among these strategies, the utilization of Fuzzy set hypothesis to take a chance with investigation appears appropriate; as such examination is profoundly abstract and connected with estimated and uncertain data (E.W.T Ngai, F.K.T Wat, 2005). In development research region, one of the uses of Fuzzy game theory is to frame a way to deal with the appraisal of the development project risk by linguistic investigation and approximation.

Kangari and Riggs (et. al 1989) bring up connecting Fuzzy set theory with risk and project risk analysis. Nonetheless, it isn't effective because of how much calculation time expected for playing out the convoluted fuzzy arithmetic operations and time for performing linguistic approximation. Chen (et. al 1996) has introduced a more proficient Fuzzy risks analysis strategy. Chen and Chen (et. al 2003) brought up the disadvantages in the comparability measure utilized in past one and proposed another strategy to decide the level of likeness between generalized Fuzzy numbers and fostered another technique to manage the issues using Fuzzy risks analysis modeling. Be that as it may, Chen and Chen (2003's) technique actually have two deficiencies: It can't accurately decide the level of likeness between generalized Fuzzy numbers in certain circumstances; and not expertise opinion dependent. In the meantime, significant quantitative models have been acquainted in writing with compute the degree of threat; which is just characterized as the pace of danger or future shortage of any framework forced by controllable or wild factors (Chavas, 2004; Doherty, 2000). A few factors, for example, likelihood of event, influence danger and capacity to fight back are presented as influencing factors on the risks. Then it is attempted to track down the numerical connection between influencing factors and the worth (level) of the risks (Li and Liao, 2007; McNeilet al., 2005). The idea of risks is impressively wide. It can contain vital, monetary, functional or some other sort of risks. Time after time this chance isn't managed sufficiently and the business has experienced horrible showing accordingly (J.H.M Tah and V.Carr, 2000).

Pejman Rezakhani, (et. al 2011) presented a Fuzzy risks analysis model linking with the model of E.W.T Ngai, F.K.T Wat, (2005). This technique is successful yet requires muddled calculations. Moreover, the technique introduced in our paper is effectively justifiable and clear. We utilized quantitative risks analysis technique applying Fuzzy number and arithmetic techniques. Nonetheless, the definitions and applications have been altered utilizing Fuzzy triangular distribution and adjusted to fit construction projects. At last, a graphical portrayal of probability and impact magnitude has been illustrated.

Accordingly, this technique can undoubtedly be applied to construction project to examine the risks through Fuzzy hypothesis.

1.1 Risks to be Considered in Construction Projects

A few specialists have recognized risk factors in development projects. In a review of Hsieh, C.H. (et. al 1996) distinguished a sum of twenty-five risk factors characterized in three categories: Internal risks factor, Project risk factors and External risk factors. Additionally, as referenced by the equivalent author(s), the main risks happened in design and building incorporate time and cost overwhelm (et. al 1985). The major factors responsible for these risks are employer or government delay, lack of information from the employer, difficulty of following instructions, conflict of interest and variation to changes. A. D. Ibrahim, A. D. F. Cost called attention to that construction projects are credited to monetary, specialized, legislative issues, force majeure and social risks which might impact the undertaking benefit [10]. Broad writing audit was done to distinguish common risks factors that might happen in construction projects [7]. This brought about distinguishing a sum of 25 factors sorted in five groups as: (i) Construction (ii) Governmental issues and agreement arrangement (iii) Monetary (iv) Plan or design and (v) Environmental. This model utilized the methodology embraced by Tah and Carr (2000) and Pezman Rezakhani et al. which include hierarchical risk breakdown structure to classify risk according to their original location impact in the project. The various leveled risk breakdown structure (HRBS) permits risk to be isolated into those categories that are connected with the administration and management of inside assets and those that are common in the outer environment [11]. External risks are moderately wild contrasted with inner risk, and they vary between projects.

1.2 Risks in Construction Projects

As indicated by PMBOK, the risk for every event can be characterized as a component of likelihood and outcome (influence). Subsequently, risk has two essential parts for a given occasion:

(i) A chance of event of that occasion
(ii) Influence (result) of the occasion happening.

Though, Risks associated with probability, some of important aspects of project uncertainty and observed project management practice cannot be explained by probability theory, like as:

- Probability assumptions are based on randomness, but project uncertainty related to the consequences of some effects are not random.
- Statistical aggregates derived from probability-based analysis become less reliable due to the unique nature of each project.
- Since uncertainty and lack of knowledge cannot be avoided in a practical scenario, it invalidates the probability theory assumptions about certain conditions and rationality in the future.
- Sources of risks in projects are not necessarily random but their level may remain unpredictable.

In this present circumstance, the hypothesis of Fuzzy sets gives a structure and offers a calculus to address these Fuzzy assertions: *"Fuzzy Sets provides a natural way of dealing with problems in which the sources of imprecision are the absence of sharply defined criteria of class membership rather than the presence of random variables"* (*Zadeh L.A., 1965).*

Fuzzy techniques have been generally utilized for tackling an incredible variety of construction issues [12]. Fuzzy logic and tools involve Fuzzy sets together to work with the course of sound judgment dissuading dubious suggestions managing etymological factors, whose values are words or sentences communicated in natural or artificial languages [13]. For example, the evaluation of the level of deviations in a given construction activity might execute linguistic values. The Fuzzy risks analysis quantitative cycle is outlined in the flowchart beneath. The degree of effect extent is the result of probability of an occasion occurring and the effect it has on the undertaking. The likelihood and effect diagrams are proposed as Fuzzy variables [24].

Impact magnitude = (possibility)*(impact)

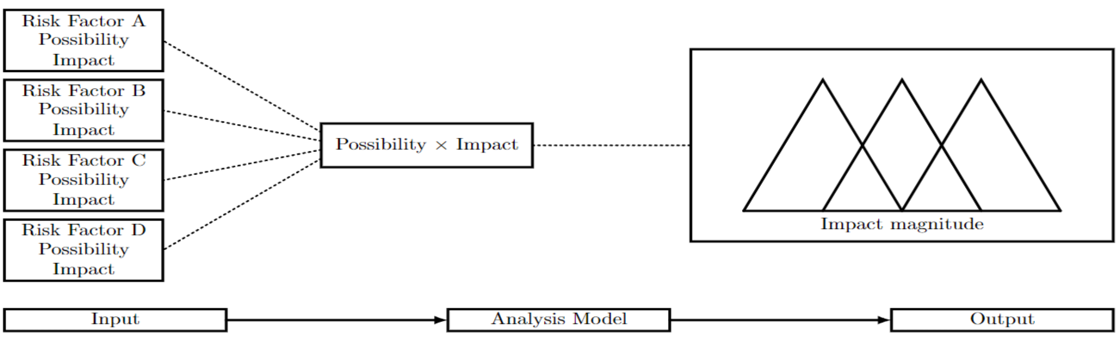

Figure 1: The Fuzzy Risks Analysis Quantitative Cycle

1.3 Fuzzy Membership Function

Traditional set hypothesis directs that a component is either an individual from a set or it is not; its membership values are characterized as 1 or 0. In Fuzzy set hypothesis this participation worth can take any genuine worth from 0 to 1 and this worth characterizes the level of membership of a given set. A membership function is a bend that characterizes how each point in the info space is mapped to a membership value (or degree of membership) somewhere in the range of 0 and 1. The main condition an enrollment capability should truly fulfill is that it should exist inside 0 and 1. In this review, the membership functions of the linguistic terms are portrayed by Triangular Fuzzy numbers on the grounds that this numbers are regularly utilized in applications, for example, Fuzzy controllers, administrative decision making, business and money, sociology, and so on. [19]. This paper considers Fuzzy membership function for construction projects as impact functions. A triangular number with membership function $\mu_A(x)$ is defined on R by

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_M-a_1} & where,\ a_1 \le x \le a_M \\ \frac{x-a_2}{a_M-a_2} & where,\ a_M \le x \le a_2 \\ 0 & otherwise \end{cases} \quad \ldots\ldots\ldots\ldots (1)$$

where, supporting interval $[a_1, a_2]$ and $(a_M, 1)$ is the peak.

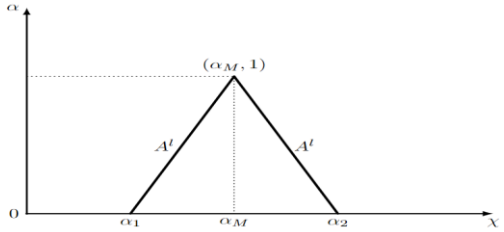

Figure 2: Fuzzy α Cuts

In application $a_M \in (a_1, a_2)$ is located at the middle of the supporting interval i.e., $a_M = \frac{a_1+a_2}{2}$. From (1) this value results central triangular fuzzy number which is as follow:

$$\mu_A(x) = \begin{cases} 2\frac{x-a_1}{a_2-a_1} & \text{for} \quad a_1 \le x \le \frac{a_1+a_2}{2} \\ 2\frac{x-a_2}{a_1-a_2} & \text{for} \quad \frac{a_1+a_2}{2} \le x \le a_2 \\ 0 & \text{otherwise} \end{cases} \quad \ldots\ldots\ldots\ldots(2)$$

Here, a_M is symmetrical with respect to the axis μ_a if in (2) $a_1 = -a,\ a_2 = a,$ Hence $a_M = 0.$ Thus, triangular number $A = (a_1, a_M, a_2)$ …… (3) then be denoted by $A = (-a, 0, a)$ and is very appropriate to express the word minimal like minimal risk. The right branch (segment) of $A = (-a, 0, a)$ if $0 \le x \le a$ can describe positive small (PS). We can denote it by $A^r = (0,0,a)$. More generally in triangular number (3) $A = (a_1, a_M, a_2)$ is suitable to represent positive large (PL) or words like high risk. Fuzzy set representation for each linguistic term is shown in the table below. Fuzzy set representation for different linguistic terms in related to construction projects [20].

Table 1: Fuzzy Set Representation for Each Linguistic Term

Possibility (x)	Impact magnitude	Impact Functions	Triangular Number	Supporting interval (x)
VU (Very Unlikely)	Minimal	$1 - 5x$	(0,0,0.2)	0% to 20%
U (Unlikely)	Low	$5x$	(0,0.2,0.4)	0% to 20%
		$2(1 - 2.5x)$		20% to 40%
M (Medium)	Moderate	$5x - 1$	(0.2,0.4,0.6)	20% to 40%
		$3 - 5x$		40% to 60%
SL (Slightly Likely)	High	$5x - 2$	(0.4,0.6,0.8)	40% to 60%
		$4 - 5x$		60% to 80%
L (Likely)	Severe	$5(x - 0.6)$	(0.6,0.8,1)	60% to 80%
		$5(1 - x)$		80% to 100%
VL (Very Likely)	Critical	$5x - 4$	(0.8,1,1)	80% to 100%

Now the Impact function of possibility and Impact magnitude is as follows:

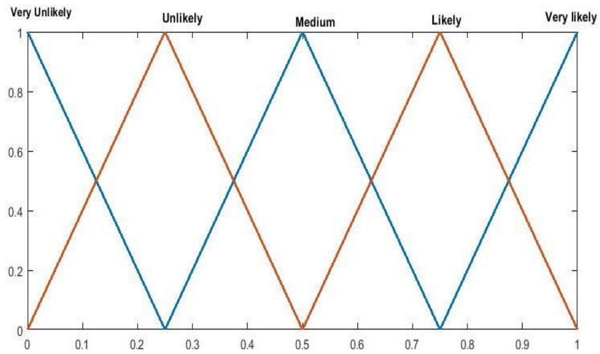

Figure 3 Membership Function of Possibility

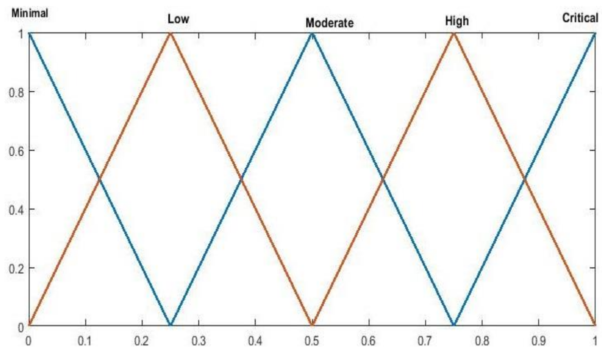

Figure 4: Membership Function of Impact Magnitude

1.4 Fuzzy Weighted Average (FWA)

In Risk and Decision analysis the most common fuzzy tool is FWA. C.H Junag, X.H. Huang, D.J. Elton (1991) first introduced this tool which is defined on the use of α -cuts as:

$$y = f(x_1, x_2, x_3, \ldots\ldots\ldots, x_n, w_1, w_2, w_3, \ldots\ldots\ldots\ldots w_n) = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \ldots\ldots\ldots + w_nx_n}{w_1 + w_2 + w_3 + \ldots\ldots\ldots + w_n}$$

$$\bar{w} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} x_i}$$

where, weighted average $\bar{w}$, x_i denotes rating for the value i and w_i is weight for value i. Here value x_i and w_i are represented by the fuzzy numbers. Schmucker used FWA to approximate numerical method known as fuzzy risk analyses. There are also some methods such as: Improved Fuzzy Weighted Average (IWFA), A Max-Min Paired Elimination

Method (MMPEM) which is a much efficient method developed by Yuh-Yuan Guh, Cheng-Chung Hon, Kuo-Milng Wang and E.S. Lee. In this paper we will use MMPEM method for calculating Fuzzy Weighted Average.

2. HOW TO PERFORM A RISK ANALYSIS

Though, there are various sorts of risk analysis, many have covering steps and targets. Each structure may likewise decide to add or change the means beneath; however, these six stages frame the most well-known course of playing out a risk analysis.

Step 1: Identify risk.
Step 2: Identify uncertainty.
Step 3: Estimate risk impact.
Step 4: Build analysis model(s).
Step 5: Analyze results.
Step 6: Implement solutions.

The chronological development of the terminology will form the model for the construction project.

2.1 Risk Identification

According to *Mehdi Tadayon et al. (2012),* four techniques are commonly used to identify risks in construction project are as follows:

I. Preparation of industrial checklists.
II. Interviews with key project participants or analysis of historical data.
III. Examining historic data from previous similar projects.
IV. Brainstorming with the project team.

In this paper, we would like to use industrial checklists for taking the values of different possibilities and impact magnitudes, which we obtained from previous historical data of similar projects. This data will be collected from field experts and professionals associated with the said projects.

2.2 Uncertainty Identification

Any project risk analysis modeling requires risk assessment process includes the possibility of the risk and its impact. Risk assessment entails significant complexities related to uncertainties and vagueness. Here first table shows customizable standard terms for measuring possibility and second table shows a customizable standard term for impact magnitude quantification. In fuzzy Weighted Average (FWA), it is necessary to define 'possibility' as the rating factor (R_i), 'impact magnitude' as the weighting factor (W_i), that corresponds to the rating factor i (*Pejman Rezakhani , et. al. 2011*)

Table 2: Customizable Standard Terms for Qualifying Possibility

Possibility	Description
VL	Expected to occur
L	Very likely to occur
SL	Slightly likely to occur
M	Likely to occur
U	Unlikely to occur
VU	Very unlikely to occur

Table 3: Customizable Standard Terms for Impact Magnitude Quantifications

Impact Magnitude	Time and Cost	Quality	Safety
Critical	>20% above target	Very poor	Injury
Severe	15% <target<20%	Poor	Safety hazard
High	10% <target<15%	Below average	Below average
Moderate	5%<target<10%	Average	Average
Low	1%<target<5%	Above average	Above average
Minimal	1%<target	OK	OK

2.3 Risk Impact Estimation

2.3.1 Fuzzy Risk Assessment and Aggregation

Expert judgment procedures have the potential for predisposition in risk recognizable proof and risk analysis, as well as in choosing risk reaction strategies. These inclinations shift on a case-by-case premise and can influence the likelihood of occurrence and consequence of occurrence estimates differently. Cognitive factors can present a predisposition or potentially commotion term [20]. Permitting more than one well-qualified assessment to evaluate the risk related with construction projects can bring about a more goal and impartial assessment. The fuzzy average operation for aggregate method that is known as the Triangular Average Formula is used to determine the mean of evaluator opinions. The triangular average formula is as follows:

Consider n inspectors/ experts and $T_i = ({O_1}^{(i)}, {M_{o1}}^{(i)}, {P_1}^{(i)})$, $i = 1, 2, 3, \ldots\ldots\ldots\ldots n$; and ${O_1}^{(i)} = optimistictime$, ${M_o}^{(i)} = mostoptimistictime$ also ${P_1}^{(i)} = pessimistictime$. By using addition to triangular numbers and division by a real number, the triangular average (mean) T_{avg} are as follow:

$$T_1 + T_2 = ({O_1}^{(1)}, {M_{o1}}^{(1)}, {P_1}^{(1)}) + ({O_2}^{(2)}, {M_{o2}}^{(2)}, {P_2}^{(2)})$$

$$= ({O_1}^{(1)} + {O_2}^{(2)}, {M_{o1}}^{(1)} + {M_{o2}}^{(2)}, {P_1}^{(1)} + {P_2}^{(2)})$$

Now, $\frac{T}{r} = \frac{1}{r}(O, M_o, P)$

$$T_{avg} = \frac{T_1 + T_2 + T_3 + \ldots\ldots + T_n}{n}$$

$$= \frac{(O^{(1)}+M_o{}^{(1)}+P^{(1)})+(O^{(2)}+M_o{}^{(2)}+P^{(2)})+\dots\dots+(O^{(n)}+M_o{}^{(n)}+P^{(n)})}{n}$$

$= \frac{(\sum_{i=1}^{n} O^{(n)}, \sum_{i=1}^{n} M_o{}^{(n)}, \sum_{i=1}^{n} P^{(n)})}{n}$ which is a triangular number,

$$T_{Avg} = (\frac{1}{n}\sum_{i=1}^{n} O^{(i)}, \frac{1}{n}\sum_{i=1}^{n} M_o{}^{(i)}, \frac{1}{n}\sum_{i=1}^{n} P^{(i)})$$

2.3.2 Fuzzy Weighted Average Computation

In this section we will discuss about the Fuzzy weighted average which is very efficient for measuring Project Risk Analysis. Here weighted average has been calculated by applying MMPEM (Y- Y GUH et al. 1996). For any α_j; $j = 1,2,3,\dots\dots\dots\dots m$, we will calculate the value of weighted average and the summarized algorithm is as follows:

1) Find the largest rating coefficient, say $a_1, a_2 \geq a_i$, $b_1, b_2 \geq b_i$and the smallest rating coefficient , say a_n,$a_n \leq a_i$, b_n, b_n,$b_n \leq b_i$, for all $i = 1,2,3,\dots\dots\dots\dots n$

2) For min$\{f_L\}$, choose c_1 as the corresponding weighting to a_1, choose $d_n$as the corresponding weighting to a_n

For max$\{f_U\}$, choose d_1 as the corresponding weighting to b_1, choose $c_n$as the corresponding

weighting to b_n.

3) Combine $a_1, a_n$and their corresponding weightingc_1, $d_n$into a new rating coefficient $a'$and the corresponding weighting w'.

For min$\{f_L\}$

$$a' = \frac{a_1 d_1 + a_n c_n}{c_n + d_1}, w' = c_n + d_1, c' = d' = w'$$

Combine b_1, b_n and their corresponding weightingc_1, d_n into a new rating coefficient b'; and the corresponding weighting w'.

For max$\{f_U\}$

$$b' = \frac{b_1 c_1 + b_n d_n}{c_1 + d_n}, w' = d_1 + c_n, c' = d' = w'$$

4) Eliminate a_1, a_n and their corresponding weighting factors c_1, d_n replace with $a'$and its corresponding weighting w'. Eliminate b_1, b_n and their corresponding weighting factors c_1, d_n replace with $b'$and its corresponding weighting w'.Merge the newly generated criteria and their weighting with the existing ones.

5) Repeat steps 1 through 4 for $(n-1)$ times, the final$[a', b']$ will be the solution for interval of α_j.

Repeat the above procedure for each α_j

2.3.3 Linguistic Approximation

It is very important to find suitable natural language expression for estimating fuzzy Risk analysis. Euclidean distance method, Successive approximation and Piecewise decomposition are three available techniques in this context. The For small set of natural language expression Euclidean method is usually applied. (*Roozbeh kangari, Leland S.Riggs, 1989*). It calculates the Euclidean distance from the given fuzzy set to each of the fuzzy sets representing the natural language expression. The successive approximation method is applied when the set is large which assumes two close primary terms, and then numerous expressions are applied to these points in order to approximate the closest natural language expression (*D.P Clements 1971; L.A Neitzel and L.J Hoffman 1980*). The piecewise decomposition method divides the linguistic variables into intervals and each interval is combined with one of the standard logical connectives to approximate the natural expression (*Y.Leung, 1980*).

In our research fuzzy triangle fitting method on impact function is applied which ensures the position of risk level on impact function. By adopting fuzzy triangle on impact function, we will take the measurement of distance from the central position of adopting triangle where $\alpha = 0$ and the closest linguistic terms on impact function for Possibility and impact magnitude on same α value where Euclidean distance method is used introduced by Ting-Yu Chen, Tai-Chun Ku and Che-Wei Tsui (et. *al 2012*). If $\tilde{T}_1 = (O_1, M_{o1}, P_1)$ and$\tilde{T}_2 = (O_2, M_{o2}, P_2)$ be two TFN's then the equation can be written as follows:

$$d(\tilde{T}_1, \tilde{T}_2) = \sqrt{\frac{1}{6}[(O_1 - O_2)^2 + 4(M_{o1} - M_{o2})^2 + (P_1 - P_2)^2};$$

where, $d(\tilde{T}_1, \tilde{T}_2)$ be the distance between the fuzzy triangular numbers $\tilde{T}_1$and$\tilde{T}_2$. This value will confirm the project productivity. The linguistic terms which one closer to the central value of adopting triangle will be regarded as that type project productivity.

2.4 Analysis Model

2.4.1 Research Framework

In this paper we will explain our proposed method by using the following six steps. Firstly, we will calculate fuzzy risk factor, then Fuzzy Weighted Average, after that we will form fuzzy triangle and represent Fuzzy number on impact function. Finally, we will analyze the calculation by applying some mathematical calculations. The flow chart shows the chronological development of the study. The numerical calculation of section 4 will follow the steps mention in the follow chart.

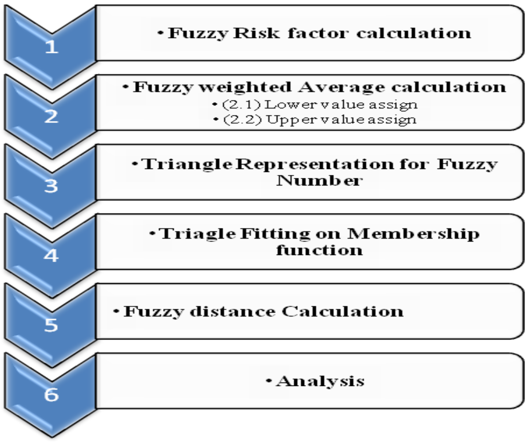

Fig 5: Flow Chart

2.4.2 *Numerical Example*

Now we will use our proposed model for a construction project where three Risk factors that can make project schedule and budget over exist bound. These factors are Plant Suitability, Weather and Resources availability, as considered the plant productivity tools. Three experts evaluate these factors are following ways:

Table 4: Evaluation of the Factors

RF	Expert Evaluation A		Expert Evaluation B		Expert Evaluation C		Expert Evaluation D	
	Possibility magnitude	Impact	Possibility magnitude	Impact	Possibility magnitude	Impact	Possibility magnitude	Impact
Suitability	Unlikely (0,0.2,0.4)	Moderate (0.2,0.4, 0.6)	Medium (0.2,0.4,0.6)	Low (0, 0.2, 0.4)	Slightly Likely (0.4,0.6,0.8)	Minimal (0,0,0.2)	Likely (0.6,0.8,1)	Severe (0.6,0.8, 1)
Weather	Medium (0.2,0.4,0.6)	Moderate (0.2,0.4, 0.6)	Unlikely (0,0.2, 0.4)	High (0.4, 0.6, 0.8)	Slightly Likely (0.4, 0.6,0.8)	Severe (0.6,0.8, 1)	Very Likely (0.8, 1,1)	High (0.4,0.6, 0.8)
Availability	Medium (0.2, 0.4,0.6)	Low (0, 0.2, 0.4)	Slightly Likely (0.4, 0.6,0.8)	Moderate (0.2,0.4, 0.6)	Likely (0.6,0.8,1)	High (0.4,0.6, 0.8)	Unlikely (0, 0.2, 0.4)	Minimal (0,0,0.2)

The step mentioned in framework of the study is presented below sequentially:

Step 1: Fuzzy Average Risk Factor Calculation:

$$T_{Avg} = (\frac{1}{n}\sum_{i=1}^{n} O^{(i)}, \frac{1}{n}\sum_{i=1}^{n} {M_o}^{(i)}, \frac{1}{n}\sum_{i=1}^{n} P^{(i)})$$

Fuzzy Average Results		
RF	Fuzzy Average of possibility (R)	Fuzzy Average of Impact magnitude (W)
Suitability	(0.3, 0.5, 0.7)	(0.2, 0.35, 0.55)
Weather	(0.35, 0.55, 0.7)	(0.4, 0.6, 0.8)
Availability	(0.3, 0.5, 0.7)	(0.15, 0.3, 0.5)

Step 2: Fuzzy Weighted Average Calculation

Calculate FWA by applying MMNE method for $\alpha = 0$ the interval of R_i and W_i where i = 1,2,3

Therefore, $[a_1 = 0.3, b_1 = 0.7][a_2 = 0.35, b_2 = 0.7][a_3 = 0.3, b_3 = 0.7]$

$[c_1 = 0.2, d_1 = 0.55][c_2 = 0.4, d_2 = 0.8][c_3 = 0.15, d_3 = 0.5]$

(2.1) Calculation Lower Bound: find $Min\{f_L\}$

Loop 1:

a_i	↓ Min 0.3	0.3	↓ Max 0.35
$[c_i, d_i]$	[0.2, 0.55] Δ	[0.15,0.5]	[0.2,0.55] Δ

$$a' = 0.3367$$

$$w' = 0.75$$

Loop 2:

a_i	↓ *Min* 0.3	↓ *Max* 0.3367
$[c_i, d_i]$	[0.15,0.5] Δ	[0.75,0.75] Δ

$$a' = 0.3305$$

$$w' = 0.9$$

Therefore, $Min\{f_L\} = 0.3305$

(2.2) Calculation Upper Bound: find $Max\{f_U\}$

Loop 3:

b_i	$\downarrow Min$ 0.7	0.7	$\downarrow Max$ 0.7
$[c_i, d_i]$	[0.2,0.55] Δ	[0.4,0.8]	[0.15,0.5] Δ

$$a' = 0.7$$
$$w' = 0.7$$

Loop 4:

b_i	$\downarrow Min$ 0.7	$\downarrow Max$ 0.7
$[c_i, d_i]$	[0.7,0.7] Δ	[0.15,0.5] Δ

$$b' = 0.7$$
$$w' = 0.85$$

Now, $Max\{f_U\} = 0.7$

Therefore, the interval for $\alpha = 0$ is (0.3305,0.7) which is each point corresponds to the end point of the triangular that represents the impact function. Now to obtain the center of the triangle α will be assigned as 1. And follow steps 1 and 2, to define Upper and Lower bound of the function. Here, the interval for $\alpha = 1$is (0.5768,0.5768) which corresponds to the center of the triangle on the impact function.

Step 3: Triangle Representation for Fuzzy Number

From step 2 we found the triangular fuzzy number (0.2097, 0.5768, 0.8405), this number corresponds a triangle on defined impact function for possibility and impact magnitude. Now by adjusting this figure on impact function for possibility and impact magnitude, we will be able to clarify the project productivity. The triangle for $\alpha = 0$and $\alpha = 1$is:

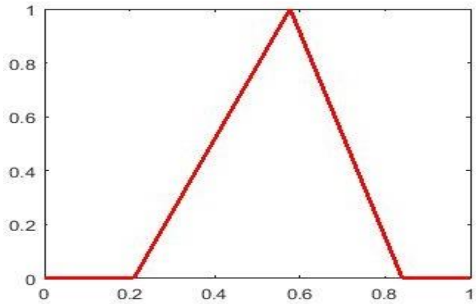

Fig 6 : Fuzzy Triangular representation

Step 4: Triangle fitting on Impact function

In this step we will fit the triangle obtained by step 3 on impact function of possibility and on impact magnitude of project assessment. This triangle is occupied by an unique space on impact function, that represents the productive region for the selected project. The figures presented below will represent the region occupied by the Fuzzy weighted average triangle on impact function.

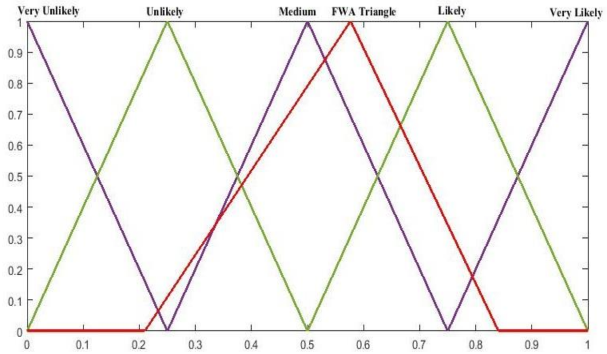

Figure 7: FWA on Possibility Membership function

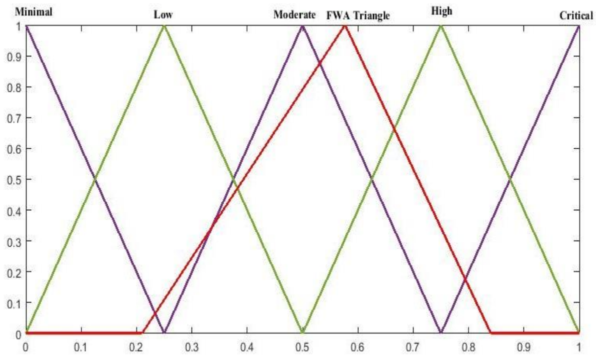

Figure 8: FWA on Impact magnitude Membership function

Step 5: Fuzzy distance Calculation
From both figures it is clear that the value of Fuzzy weighted average lies in between 0.2097 to 0.8405. In this range at point 0.5768, the FWA triangle gets the high value 1. Adopting the fuzzy triangle (0.2097,0.5768,0.8405) on impact function, it is clearly observed that, the closest two linguistic terms are medium and likely for possibility whereas moderate and high for impact magnitude. Here the triangular fuzzy number formed on impact function for Linguistic term Medium or Moderate is (0.25, 0.5, 0.75) and for linguistic term Likely or High is (0.5, 0.75, 1).

Now, $d(A.Triangle, Medium/Moderater) = 0.074$and

$d(A.Triangle, Likely/High) = 0.195$

Step 6: Analysis
Here the fuzzy distance between adopting triangle and Medium or Moderate triangle on impact function is 0.074 and this distance for Likely or High is 0.195. Therefore, this optimum value is close to Medium on possibility impact function, on the contrary, in impact magnitude impact function is close to Moderate value. Therefore, in the analysis, the project productivity is Medium on possibility and moderate on Impact magnitude. In precisely we can say that the project productivity is moderate on presented data and observation.

2.5 Result Analysis

Due to some vague or unknown factors, it is challenging to calculate the cost and time of any construction project accurately. Therefore, it requires projections. In this case, quantitative risk analysis modeling is appropriate while project managers can take the opinions of experts and associated professionals, which when considered in the proposed model can estimate the impact magnitude of possible risks during the project. This allows us to produce a triangle representation of fuzzy number for impact magnitude. Analyzing this triangle will help to predict the productivity and likelihood of success of a project.

2.6 Solution Implementation

Depending on the result analysis of our model, product managers can take important decisions on the future of the project. While a project should proceed as usual when the model predicts high productivity and high likelihood of success, project managers can take important steps to minimize the possibility and mitigate the impact of risks when the model predicts otherwise.

3. CONCLUSIONS

In this paper the method presented is effective and applicable for any type of construction project comparable to any other available method. This method is easy to understand with a short and straightforward calculation procedure. Therefore, anyone having minimum knowledge about a project can use this method.

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