International Journal of Computer Applications (0975 – 8887)
Volume 186 – No.25, June 2024

PAPR Reduction of Single Carrier FDMA Signals using Efficient SINC Pulse Shaping

M.V.R. Vittal
Department of Electronics & Communication Engineering,
G.Pulla Reddy Engineering college (Autonomous) ,
Kurnool, Andhra Pradesh state, India

ABSTRACT
Orthogonal frequency division multiplexing (OFDM) system is one of the important multicarrier modulation techniques that satisfies the need for high spectral efficiency and good quality of service. It is an efficient candidate for supporting the current 5G technology and aims at achieving high data rate in wireless communications. Single carrier FDMA (SC-FDMA) system has equal performance and the complexity matches to that of OFDM. But high Peak to average power ratio (PAPR) is an issue that increases the complexity of analog to digital converters. In this paper, Localized FDMA(LFDMA), Distributed FDMA(DFDMA) and Interleaved FDMA(IFDMA) schemes are discussed and the PAPR analysis of SC-FDMA signals using Raised cosine and Sinc pulse shapings is carried out and their performance is compared.

Keywords
Peak to Average Power ratio (PAPR), SC-FDMA, Pulse shaping, Discrete Fourier Transform (DFT) spreading, subcarriers, Complementary cumulative distribution function (CCDF), Raised cosine (RC) pulse shaping, Sinc pulse shaping.

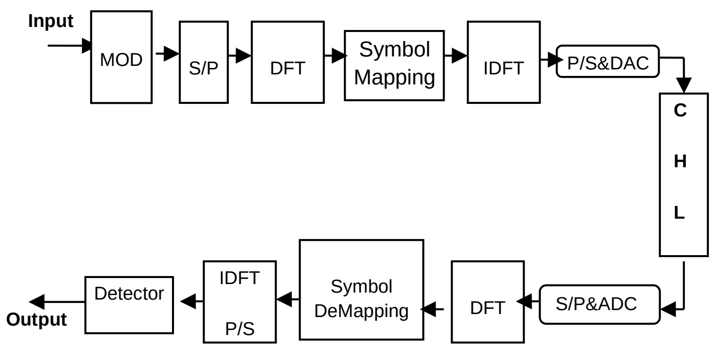
1. INTRODUCTION
The main goal of the fourth and fifth generation wireless systems is to provide high data rates and a wider range of services such as voice communications, videoconferencing, and high-speed internet access. Orthogonal frequency division multiple access (OFDMA) is a multiple user version of OFDM that serves as a strong candidate to achieve these services. Multiple access in this scheme is obtained by allocating subsets of carriers to different users. SC-FDMA systems exhibit similar performance compared to OFDMA systems. But high PAPR is a major technical challenge in SC-FDMA systems that reduces the efficiency of RF power amplifiers. The basic advantage of SC-FDMA systems is that its PAPR is less compared to OFDMA systems because of the availability of single carrier. The decrease in PAPR of SC-FDMA signals using DFT for FBMC signals is analyzed in [1]. The analysis of PAPR vs CCDF using DFT spreading techniques with RC pulse shaping for various roll of factors is performed in [4]. Reduced complexity PAPR reduction techniques for SC-FDMA have been cited in [6]. PAPR reduction schemes for LTE-A SC-FDMA systems are discussed in [8]. But PAPR analysis of DFT spreading techniques using Sinc pulse shaping is not carried out in [4] which is the consequence of high PAPR. In this paper, the PAPR analysis of SC-FDMA systems using LFDMA, DFDMA and IFDMA techniques with both raised cosine pulse shaping and the Sinc pulse shapings is carried out and their comparative analysis is done. It is observed that Sinc pulse shaping has the better performance in decreasing PAPR of SC-FDMA signals.

This paper is described as follows: The basic SC-FDMA system is discussed and PAPR is formulated in Section 2, Section 3 describes the DFT spreading techniques and explains about the Raised cosine and Sinc pulse shapings, Simulation results are presented in Section 4 and Conclusion is discussed in Section 5.

2. BASIC SC-FDMA SYSTEM
The block wise description of the SC-FDMA system is depicted in Figure 1. Symbol mapping is done in Transmitter after the FFT. Then the IFFT operation converts data symbols into the time domain. Subcarrier demapping is performed at the receiver post the DFT operation. Then the IDFT converts the samples into time domain. These samples are passed through the detector to obtain the original signal.

The operations of DFT and IDFT are performed separately at the transmitter and receiver of SC-FDMA system. The output of the transmitter is passed through a wireless channel that is carefully modelled. Digital modulation and demodulation schemes are employed.

2.1. Concept of PAPR
Peak to Average Power Ratio (PAPR) results as a consequence of summing of carriers together. The maximum peak power varies directly as the number of carriers in the system. The scalar relationship is lost following the linear region and the amplifier proceeds into saturation region. The working of amplifiers in the saturation region results in distortion and it is is a major limitation.



P/S-Parallel to Serial , S/P-Serial to parallel, CHL-channel

Figure 1. Block diagram of SC-FDMA system

PAPR is expressed as

$$PAPR = \frac{\sum_{n=0}^{N-1} max\{x^2(n)\}}{P_{av}\{x(n)\}} \qquad (1)$$

x(n) is the information signal
max{x^2(n)} denotes the maximum power of signal
N is the number of symbols containing data

$$p_{av}\{x(n)\} = \frac{1}{N}\sum_{n=0}^{N-1} E\{x^2(n)\}$$

(2)
indicates the average power in signal
$E\{x^2(n)\}$ in (2) is the mean square value of x(n) and 'E' indicates expectation.

3. PROPOSED WORK

In DFT spreading technique, the subcarrier allotment is performed to users by the three schemes that are named as Localized FDMA(LFDMA), Distributed FDMA(DFDMA) and Interleaved FDMA(IFDMA). In LFDMA scheme, M number of subcarriers are consecutively allotted in a total of N subcarriers and the unused subcarriers are consecutively occupied with zeros. In DFDMA scheme allotment of M outputs is done over the entire band of N subcarriers and the rest of unused subcarriers are occupied with zeroes.
In IFDMA scheme, allotment of M DFT outputs is performed over N subcarriers with equidistance. Here new parameter known as spreading factor as L=N/M is defined and M is the number of DFT outputs distributed.
The allotment of subcarriers by these three schemes is depicted in Figure 2. The total number of subcarriers is N=12. Here it is assumed that 4 DFT outputs are distributed. (M=4). The spreading factor is L=3. X(0), X(1),X(2),X(3) denote the DFT outputs distributed using LFDMA, DFDMA and IFDMA schemes.
In the subsequent sections, the expressions for IFFT outputs for LFDMA, IFDMA and DFDMA are derived. The IFFT output in each scheme is extracted after the DFT and subcarrier mapping operations. The importance of IFFT outputs is that it is the time domain signal depending on certain factors like multiplication, rotation in phase, complex factor, etc.

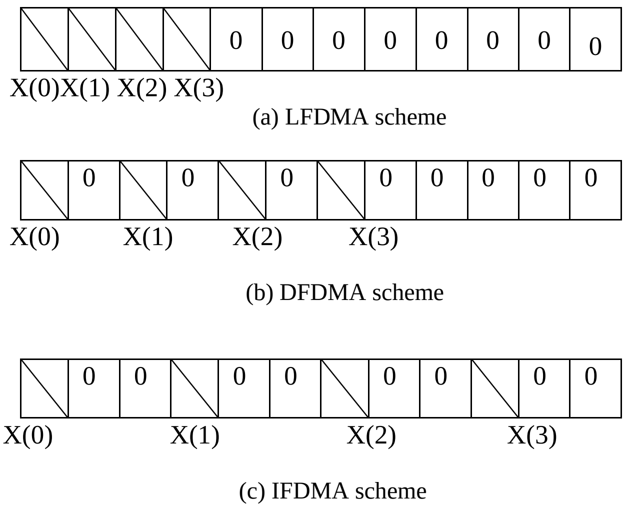


Figure 2. DFT Spreading Techniques with distribution of outputs of DFT (N=12, M=4, L=3)

3.1.DFT SPREADING TECHNIQUES

3.1.1. LFDMA scheme

A total amount of N subcarriers is considered here and let M be the successive DFT outputs allotted with the unused subcarriers occupied with zeroes.
Consider $n = Lm + l$, here L is the Spreading factor and $0 \le l \le L-1,\ 0 \le m \le M-1$

The output of LFDMA is given by

$$X_{LFD}(k) = X(k), k = 0,1,\ldots.(M-1)$$

(3)

$$= 0 \qquad \text{otherwise}$$

and the IFFT output sequence is specified as

$$x_{LFD}(n) = x_{LFD}(Lm + l), \qquad 0 \le n \le N-1$$

(4)

Now $x_{LFD}(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_{LFD}(k) e^{\frac{j2\Pi kn}{N}}$ (5)

$$= \frac{1}{LM}\sum_{k=0}^{M-1} X(k) e^{\frac{j2\Pi k(Lm+l)}{LM}}$$

(6)
If l =0, then from (6),

$$x_{LFD}(n) = \frac{1}{LM}\sum_{k=0}^{M-1} X(k) e^{\frac{j2\Pi mk}{M}}$$

$$= \frac{1}{L} x(m)$$

(7) If $l \neq 0$, then from (6)

$$x_{LFD}(n) = x_{LFD}(Lm + l)$$

$$= \frac{1}{LM}\sum_{k=0}^{M-1} X(k) e^{\frac{j2\Pi(Lm+l)k}{LM}}$$

(8)
DFT of x(n) can be written as

$$X(k) = \sum_{r=0}^{M-1} x(r) e^{\frac{-j2\Pi kr}{M}}$$

$$= \frac{1}{LM}\sum_{k=0}^{M-1}\sum_{r=0}^{M-1} x(r) e^{\frac{-j2\Pi rk}{M}}\ e^{j2\Pi\{\frac{m}{M}+\frac{l}{LM}\}k}$$

(9)

$$= \frac{1}{LM}\sum_{k=0}^{M-1}\sum_{r=0}^{M-1} x(r) e^{j2\Pi\{\frac{m-r}{M}+\frac{l}{LM}\}k}$$

(10)

On simplifying (10), the expression is

$$x_{LFD}(n) = \frac{1}{LM}\sum_{r=0}^{M-1} \frac{x(r)}{1-e^{j2\Pi\{\frac{m-r}{M}+\frac{l}{LM}\}}}$$

(11)
It is observed from above that the IFFT output for LFDMA is dependent on the scaled version of the input signal by $1/L$ and with a complex weighting factor.

3.1.2. IFDMA scheme

Here the factor $k = pL$ is considered where p is the integer that denotes the allotted DFT output and L signifies the spreading factor for IFDMA.

The output of DFT for IFDMA is

$$X_{IFD}(k) = X(k/L), k = Lp$$

(12)
Here p= 0,1,……(M-1)

The IFFT output is $x_{IFD}(n)$ where n= $Ml+m$

Now $$x_{IFD}(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_{IFD}(k)e^{\frac{j2\Pi kN}{N}}$$ (13)

Putting $k = Lp, N = LM$ in (13),

$$x_{IFD}(n) = \frac{1}{ML}\sum_{p=0}^{M-1} X(k/L)e^{\frac{j2\Pi Lpn}{ML}}$$ (14)

$$= \frac{1}{ML}\sum_{p=0}^{M-1} X(k)e^{\frac{j2\Pi pn}{M}}$$ (15)

Substituting $n = Ml + m$ in (15),

$$x_{IFD}(n) = \frac{1}{ML}\sum_{p=0}^{M-1} X(p)e^{\frac{j2\Pi(lM+m)p}{M}}$$ (16)

Simplifying (16),

$$x_{IFD}(n) = \frac{1}{L}\frac{1}{M}\sum_{p=0}^{M-1} X(p)e^{\frac{j2\Pi mp}{M}}$$
$$= \frac{1}{L}IDFT\{X(p)\}$$
$$= \frac{1}{L}x(m)$$ (17)

It is evident from (17) that the output of IFDMA is the information signal x(m) scaled by a factor of 1/L and L is the spreading factor. Next discussion is done about the distributed FDMA scheme.

3.1.3. DFDMA scheme

The output of DFDMA following subcarrier mapping is

$$X_{DFD}(k) = X(p), p = 0,1,\dots.(M-1)$$ (18)

Here $k = Lp + r, r = 0,1,\dots(L-1)$

Then

$$x_{DFD}(n) = x_{DFD}(LM + l)$$ (19)

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_{DFD}(k)e^{\frac{j2\Pi kn}{N}}$$ (20)

Putting k=Lp+r in (20),

$$x_{DFD}(n) = \frac{1}{ML}\sum_{p=0}^{M-1} X(p)e^{\frac{j2\Pi(Lp+r)n}{N}}$$ (21)

$$= \frac{1}{ML}\sum_{p=0}^{M-1} X(p)e^{\frac{j2\Pi Lpn}{N}}e^{\frac{j2\Pi rn}{N}}$$ (22)

Putting $n = LM + l$ and $N = LM$ in (22),

$$x_{DFD}(n) = \frac{1}{ML}\sum_{p=0}^{M-1} X(p)e^{\frac{j2\Pi(LM+l)p}{M}}e^{\frac{j2\Pi rn}{N}}$$ (23)

$$= \frac{1}{L}e^{\frac{j2\Pi rn}{N}}\frac{1}{M}$$
$$= \frac{1}{L}e^{\frac{j2\Pi rn}{N}}\frac{1}{M}IDFT\{X(p)\}$$

The output of IFFT is given as

$$x_{DFD}(n) = \frac{1}{L}e^{\frac{j2\Pi rn}{N}}x(m)$$

(24) It is obvious from (24) that the IFFT output of DFDMA is the phase rotation of the input signal x(m) and with a multiplication factor.

3.2. Pulse shaping to reduce PAPR

High PAPR is the main limitation in SC-FDMA systems and the objective is to decrease PAPR to a minimum. Decrease in PAPR in SC-FDMA systems can be effectively done by the above DFT spreading techniques along with pulse shaping. With pulse shaping, PAPR can be reduced by decreasing the peak power of the transmitted signal and bandwidth efficiency of the system need not be affected. The concept of pulse shaping for the SC-FDMA system is discussed below.

An SC-FDMA system is considered where the information signal is x(n) for 0≤n≤N-1 to be modulated and let f_c be the frequency of carrier. X(k) signifies the DFT of x(n) for 0≤k≤N-1. Following subcarrier mapping let X(q) signify the output for 0≤q≤M-1 and let x(p) indicate the output after the IFFT operation of X(q) for 0≤p≤M-1. The complex bandpass signal of SC-FDMA system is specified as

$$x(t) = e^{j2\Pi f_c t}\sum_{p=0}^{M-1} x(p)s(t-pT) \quad (25)$$

In above equation, s(t) is the baseband pulse. Now, mainly two prominent pulse shaping techniques that are named as the Raised cosine pulse shaping and Sinc pulse shaping are focused and their performance is compared.

3.2.1. Pulse Shaping using Raised Cosine pulse

The raised cosine pulse is prominently employed in Wireless communications. It is defined as

$$s_{RC}(t) = sin\,c\,(\frac{t}{T})\frac{cos(\frac{\Pi\beta t}{T})}{1-\frac{4\beta^2t^2}{T^2}}$$ (26)

Here β is the roll-off factor which varies from 0 to1 and T is the duration of each symbol.

With Raised cosine pulse shaping, (25) can be written as

$$x(t) = e^{j2\Pi f_c t}\sum_{p=0}^{M-1} x(p)s_{RC}(t-pT)$$ (27)

3.2.2. Pulse Shaping using Sinc pulse

An efficient pulse used for pulse shaping to reduce the PAPR of SC-FDMA signals is the Sinc pulse specified as

$$p(t) = \frac{\sin(\pi t/T_s)}{\pi t/T_s} \quad (28)$$

Here $p(t)$ is the time domain signal of the Sinc pulse where T_s is the symbol duration and related to Symbol rate as

$$F_s = \frac{1}{T_s}$$

(29) The Sinc pulse has a rectangular shape in frequency domain and avoids inter symbol interference (ISI) with minimum amount of bandwidth. It exhibits pulse area preservation properties that helps to use in pulse shaping.

Owing to these properties, Sinc pulse is employed in wireless communications for pulse shaping.

4. RESULTS

Simulations are carried out and the results are graphically plotted to generate the Complementary Cumulative distribution functions (CCDF's) of the three DFT spreading techniques namely LFDMA, IFDMA and DFDMA.

4.1. Simulation Parameters

The number of subcarriers N is 256, the size of data block is 64, the block size is 8000, the roll off factor for the Raised cosine pulse is set to 0.22, the duration of symbol for Sinc pulse is taken as 0.5ms and the oversampling factor is 8. Modulation taken is 16 QAM. MATLAB software is used to obtain the Simulation results with and without Pulse shaping.

Table 1. List of Simulation parameters

S.No	Simulation Parameters	Value
1.	Number of subcarriers	256
2.	Size of data block	64
3.	Block size	8000
4.	Roll off factor for RC pulse	0.22
5.	Duration of symbol for Sinc pulse	0.5 ms
6.	Modulation	16 QAM

Table 1 shows the list of Simulation Parameters.

4.2 Simulation Results and Discussion

4.2.1. PAPR performance without pulse shaping

Figure 3 shows the comparative performance of three DFT spreading techniques namely LFDMA, DFDMA and IFDMA schemes when no pulse shaping is applied. From the figure, it is observed that the PAPR of the three schemes are almost nearly equal. There is a slight reduction in PAPR of IFDMA scheme compared to that of LFDMA and DFDMA schemes. But there is no significant reduction in PAPR when no pulse shaping is applied.

4.2.2. PAPR performance with RC and Sinc pulse shapings

The PAPR performance of the three schemes including LFDMA, DFDMA and IFDMA schemes with Raised cosine (RC) and Sinc pulse shapings is shown in Figure 4. The graph shows Peak to Average power ratio on X-axis versus Complementary cumulative distribution function on Y-axis. From the figure, it is evident that both LFDMA and DFDMA schemes with Sinc pulse shaping achieve lower PAPR compared to that of the two schemes with RC pulse shaping.

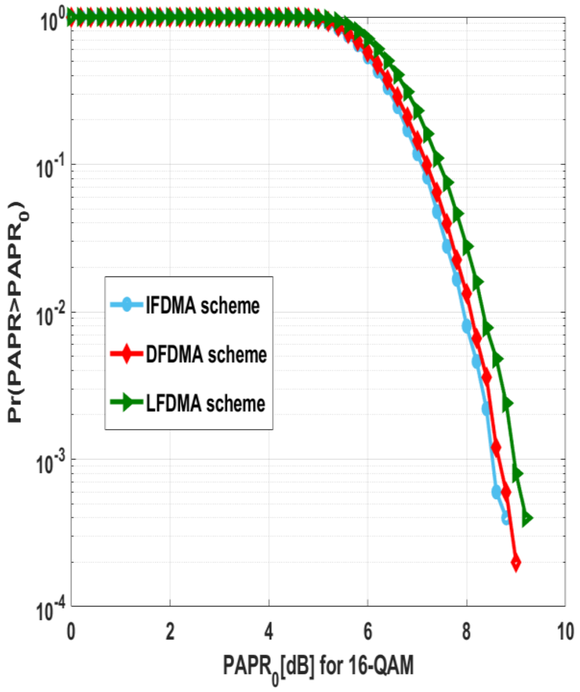


Figure 3. PAPR performance comparison of DFT spreading Schemes with no applied pulse shaping

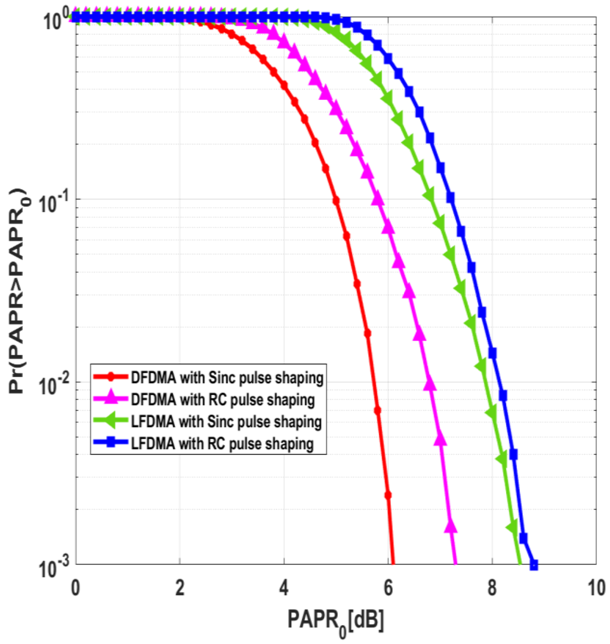


Figure 4. PAPR performance comparison of LFDMA and DFDMA schemes with RC and Sinc pulse shaping

4.2.3. It is also evident from the figure that DFDMA scheme attains more reduction in PAPR compared to that of LFDMA scheme. At a CCDF of 10^{-3}, the value of PAPR is about 6.2 dB with Sinc pulse shaping that is nearly 0.9 dB lesser than that with RC pulse shaping. Figure 5 shows the comparative performance of IFDMA scheme with Raised cosine and Sinc pulse shapings. It is observed that IFDMA scheme with Sinc pulse shaping attains lower PAPR compared to that with RC pulse shaping. At a CCDF value of 10^{-3}, the value of PAPR is about 6 dB with Sinc pulse shaping that is nearly 1 dB lesser than that with RC pulse shaping.

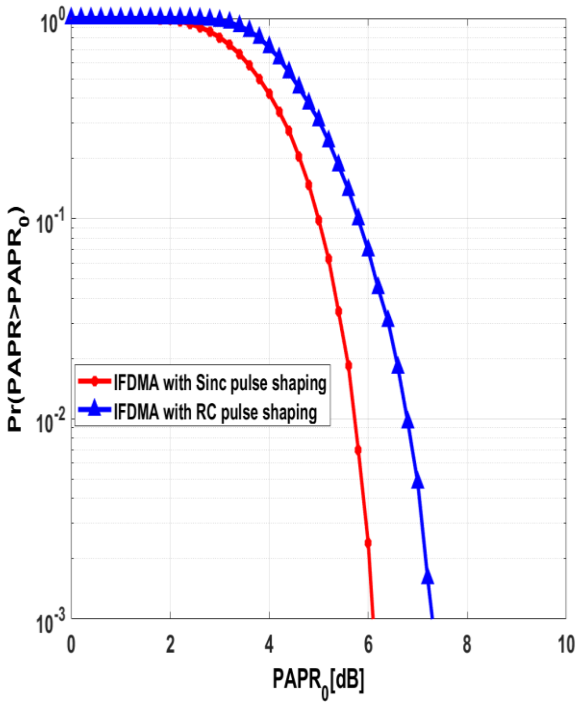


Figure 5. Comparison of PAPR performance of IFDMA scheme with RC and Sinc pulse shaping

4.2.4. The values of Peak to Average power ratio that employ the three schemes LFDMA, DFDMA and IFDMA with Raised cosine and Sinc pulse shapings is shown in Table 2. These values are obtained from the Simulation results graphs of Figure 4 and Figure 5 respectively.

Table 2. Performance comparison of PAPR using LFDMA, DFDMA and IFDMA schemes

	PAPR (dB) for LFDMA	**PAPR (dB) for DFDMA**	**PAPR (dB) for IFDMA**
RC pulse shaping	9.0	7.1	7.0
Sinc pulse Shaping	8.4	6.2	6.0

5. CONCLUSION

In this paper, the PAPR analysis of SC- FDMA signals using the three DFT spreading techniques LFDMA, IFDMA and DFDMA is considered and their PAPR performance is compared using Raised cosine and Sinc pulse shapings. It is observed that the three techniques using Sinc pulse shaping have lower PAPR compared to that using RC pulse shaping. Among the three, the IFDMA technique with Sinc pulse shaping has the lowest PAPR. The PAPR performance of DFDMA and IFDMA schemes is nearly same.

So, it is concluded that the implementation of DFDMA and IFDMA schemes with Sinc pulse shaping will be more effective as far as the lower PAPR performance is concerned. LFDMA scheme is used where the allotment of outputs to the subcarriers is simpler. As Sinc pulse uses minimum bandwidth and avoids ISI, SC-FDMA systems can make use of DFT spreading techniques with Sinc pulse shaping to improve the efficiency and reduce PAPR effectively. SC-FDMA system finds its wide applications in Wireless and Cellular networks, Long term evolution (LTE), LTE advanced (LTE-A), fifth generation (5G) wireless communication systems for high data rate requirements.

SC-FDMA systems with Sinc pulse shaping can also be implemented in massive MIMO systems and NOMA systems where the reduction of PAPR is much desirable. As the research is going on 6G Wireless communication systems that satisfy the need for huge data rates, SC-FDMA systems can be employed in 6G wireless systems in future to reduce the Peak to Average power ratio and to enhance the performance of the systems.

6. REFERENCES

[1] M.A.Aboul Dahab (2019) "Generalized DFT for FBMC PAPR reduction", IEEE Access Journal, 7, 81730-81740, June 2019.

[2] Kun Wu, G.Ren (2015), "PAPR reduction of SC-FDMA signals using optimized additive predistortion", IEEE Communication letters, 19(8), 1446-1449, Aug.2015.

[3] J.Ji, G.Ren (2015) "PAPR reduction of SC-FDMA signals via probabilistic Pulse shaping", IEEE transactions on Vehicular Technology, 64(9), 3999-4008, Sep. 2015.

[4] H. G. Myung, J. Lim and D. J. Goodman (2006), "Peak-to Average Power Ratio of Single Carrier FDMA Signals with Pulse Shaping,"17th IEEE International Symposum on PIMRC,1-5, Sep. 2006.

[5] G.Hyuang, A.Mix, S.Armour(2007), "Impact of Radio Resource Allocation and pulse shaping on PAPR of SC-FDMA signals", 18th IEEE IEEE International symposium on PIMRC, 2007.

[6] P.Yen and H.Min (2012), " Low Complexity PAPR reduction methods for carrier aggregated OFDMA and SC-FDMA systems", EURASIP Journal on Wireless Comn & Networking, 1-13, May 2012.

[7] Z.Soud and B.Ridha (2011), "SOCP Approach for Reducing PAPR System SC-FDMA in Uplink via TR scheme" Jounal of Computer Networks and Communication, 3(6),157-168, Nov.2011.

[8] W.Radi, H.Badiawy (2011), "PAPR reduction Techniques for Long term Evolution SC-FDMA system" Journal *of Adv. Engg.Sciences & Technologies,* 6(2), 230-236, June 2011.

[9] Sangeetha S, Rajaram M (2011), "PAPR reduction in OFDM for SC-FDM channels", IEEE International Conference .on ICSCCN,6-9, July 2011.

[10] Y.Li, I.J.Li,J.Kim (2009), "PAPR analysis of OFDMA and SC-FDMA in the uplink of a Mobile Commn system" Journal of the Korean Institute of Electromagnetic Engg & Science, 9(1),17-24, Mar.2009.

[11] H. G. Myung, J. Lim and D. J. Goodman (2006) "Single Carrier FDMA for Uplink Wireless Transmission," IEEE Vehicular Technology Magazine., 1(6), 30-38, Sep. 2006.

[12] T. S. Rappaport, "Wireless Communications: Principles and Practice", Second Edition. Prentice Hall, 2004.