International Journal of Computer Applications (0975 – 8887)
Volume 186 – No.25, June 2024

Edge Rotations, Edge Jumps and its Effect on Certain Graph Parameters

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ABSTRACT

Let S be a set of graphs on which a measure of distance (a metric) has been defined. The distance graph $D(S)$ of S is that graph with vertex set S, such that two vertices(graphs) G and H are adjan- cent if and only if the distance between G and H is one. A graph H is obtained from a graph G by an edge rotation if G contains three distinct vertices u,v and w such that $uv \in E(G)$, $uw \notin E(G)$ and $H \cong G - uv + uw$. In this case, G is transformed into H by "rotating" the edge uv of G into uw. A graph H is obtained from a graph G by an edge jump if G contains four distinct vertices u, v, w and x such that $uv \in E(G)$, $wx \notin E(G)$ and $H \cong G - uv + wx$.

In this paper, I investigate the effect of the above mentioned edge operations viz., rotation and jump on certain graph parameters. I investigate rotations on DDR graphs, rotations on cycles, paths, Eulerian graphs, eccentric digraphs and the planarity property of the connected graph post edge rotation. We also present an algorithm that generates all rotation distance graphs at distance one.

General Terms

2000 *Mathematics Subject Classification.* Primary 05C12, secondary 05C75.

Keywords

Edge rotations, Edge jumps, edge rotation distance graphs, r -distance graph, edge jump distance graph, j -distance graph, planar graph, rotation distance graph, jump distance graph, eulerian graph, self-centered graph, cycle, path, DDR graph.

1. INTRODUCTION

Unless mentioned otherwise, for terminology and notation the reader may refer Buckley and Harary [3] and Chartrand and Zhang [9], new ones will be introduced as and when found necessary.

In this paper, by a graph G, I mean a simple, undirected, connected graph without self-loops. The order and size are respectively the number of vertices denoted by n and the number of edges denoted by m.

The distance d(u, v) between any two vertices u and v, of G, is the length of a shortest path between u and v. The eccentricity e(u) of a vertex u is the distance to a farthest vertex from u. The maximum and the minimum eccentricity amongst the vertices of G are respectively called the diameter diam(G) and radius rad(G). If d(u, v) = e(u), $(v \neq u)$ then we say that v is an eccentric vertex of u.

Definition 1.1. The distance degree sequence (dds) of a vertex v in a graph G = (V , E) is the list of number of vertices at distance 1, 2, ...,e(v), in that order, where e(v) denotes the eccentricity of v. Thus the sequence $(d_{i0}, d_{i1}, d_{i2}, . d_{ij})$, is the distance degree sequence (dds) of the vertex v_i in G where d_{ij} denotes the number of vertices at distance j from v_i.

Several distances between graphs works with respect to transformations. In this paper I consider two elementary transformations namely edge rotations and edge jump.

The transformation between graphs here is completely based on edge operations like edge jump and edge rotation. To perform them the basic idea is to usually consider two graphs G and H having the same order and the same size. V. Balaz et al. in [1] showed that a graph H is said to be obtained from G by an edge move if G contains vertices u, v, w and x (not necessarily distinct) such that $uv \in E(G)$, $wx \notin E(G)$ and $H \cong G - uv + wx$. A graph G is $m-$transformed (or move transformed) into a graph H if H is obtained from G by a sequence of edge moves. In [6] Chartrand et al. showed that a graph H is said to be obtained from G by an edge rotation if G contains distinct vertices u, v and w such that $uv \in E(G)$ and $uw \notin E(G)$ and $H \cong G - uv + uw$. The rotation distance between graphs G and H is denoted by $d_r(G, H)$, if there exists a sequence of graphs $G_1, G_2, \ldots, G_{k-1}$ such that G_1 is obtained by an edge rotation on G, and for each $1 \leq i \leq k$, G_{i+1} is obtained by an edge rotation on G_i, with H obtained from G_{k-1} by one edge rotation. In this case we denote the rotation distance from G to H as $d_r(G, H)$ and it is equal to k. We shall denote the rotation operated graph as G^r for convenience.

Definition 1. [6] Let $S = G_1, G_2, ..., G_k$ be a set of graphs all of the same order and the same size. Then the rotation distance graph D(S) of S has S as its vertex set and vertices (graphs) G_i and G_j are adjacent if $d_r(G_i,G_j) = 1$, where $d_r(G_i, G_j)$ is the rotation distance between G_i and G_j.

Definition 1. [6] Let $S = G_1, G_2, ..., G_k$ be a set of graphs all of the same order and the same size. Then the rotation distance graph D(S) of S has S as its vertex set and vertices (graphs) G_i and G_j are adjacent if $d_r(G_i,G_j) = 1$, where $d_r(G_i, G_j)$ is the rotation distance between G_i and G_j.

A graph G is a edge rotation distance graph(ERDG) (or r -distance graph) if $G \cong D(S)$ for some set S of graphs.

Definition 2. [8] Let $S = G_1, G_2, ..., G_k$ be a set of graphs all of the same order and the same size (atleast 5). Then the jump distance graph $D_j(S)$ of S has S as its vertex set and vertices (graphs) G_i and G_j are adjacent if $d_j(G_i,G_j) = 1$, where $d_j(G_i, G_j)$ is the jump distance between G_i and G_j. A graph G is a jump rotation distance graph(EJDG) (or j - distance graph) if G $\cong$ D(S) for some set S of graphs.

In 1990, Chartrand et al. [7] showed that the cycles, the complete bipartite graphs $K_{3,3}$, and $K_{2,n}$ $(n \geq 1)$ are edge rotation distance graphs (ERDG). Later in 1997, Jarrett [20] gave a different proof technique and thus showed complete graphs, trees and wheel ($W_{1,n}$) belong to the class of edge rotation distance graphs. It was shown that the complete bipar- tite graph $K_{m,n}$ ($3 \leq m \leq n$) is a edge rotation distance graph. In [17], [18], [19] Chitra et al. characterized few graphs to be ERDG and EJDG, and also showed that the Generalized Petersen Graph, $G_p(n, 1)$, the generalized star, $K_{(1,n)}$ are edge rotation distance graphs. In [18] Chitra et al. showed that the Generalized Petersen graph, $G_p(n, 1)$ and the general- ized star, $K_{(1,n)}$ are edge jump distance graphs. They further extended the work in [19] in showing the Ladder graph, Triangular snake, Quadrilateral Snake,Double triangular snake, Double Quadrilateral snake, Alternate triangular snake, Alternate quadrilateral snake are all edge rotation distance graphs (ERDG). Similar study is undertaken for jump distance, and several families of graphs have been shown to be edge jump distance graphs (EJDG). Many results are due to Jarrett [20] and Chartrand et al. [8].

In this paper I find rotations on randomly generated Eulerian graphs from 0(zero) vertices. Also rotations on Distance degree regular (DDR) graphs and few results on some standard graphs with respect to edge rotation and edge jump and their effect on certain graph parameters. I present two conjectures with respect to edge rotation and also show Eulerian graph generated from 0 vertices is Edge rotation distance graph (ERDG).

2. EDGE ROTATIONS AND EDGE JUMP ON DISTANCE DEGREE REGULAR GRAPHS

The concept of distance degree regular (DDR) graphs was introduced by Bloom et al.[5] as the graphs for which all vertices have the same distance degree sequence. The study of DDR graphs was undertaken further by Bloom et al.[4], Halberstam et al.[12] and Huilgol et al. [13] ,[14] , [15]. In [13] Huilgol et al. have characterized DDR graphs of diameter three of certain extreme regularities. In the same paper they have given a general construction of a diameter three DDR graphs. But till date the characterizations for DDR graphs do not exist. Huilgol et al. [16] have defined Almost Distance Degree Regular graphs (or ADDR in short). Almost Distance Degree Regular (ADDR) graphs is defined as a graph G of order n if $n - 1$ vertices have the same distance degree sequences and one vertex with different distance degree sequence. Here I consider the graphs from [13], and operate rotations on them.

3. MAIN RESULTS

Theorem 2.1. If G is a DDR graph of diameter 3 and regularity $d = p - 6$, then rotation of any edge does not result in a DDR/ADDR graph.

Proof. The DDR graphs were characterized by Huilgol et al. in [13] having diameter three and regularity , $d = p - 6$. Applying the definition of rotations, we start considering the edges from the set A to the set B. Taking one edge rotation, results in a non DDR graph. Hence we consider two rotations. Also we consider rotation with in the set A and rotation between the set A and the set B. First, let us consider the rotation(with in set A) from the vertex u_1 to the say another vertex u_i from the same set which is at distance two. By performing this, the degrees of atmost two vertices change, thus resulting in a vertex of higher degree and one vertex of lesser degree. To avoid this we make one more rotation, to the vertex which lost one degree in the earlier edge operation. The second case would be considering rotation between the set of vertices from the set A to the set B. Here again we consider two rotations in order to balance the degree of all the vertices. Thus, on repeating such several edge operation between the two sets we find that the distance degree sequence is not uniform through out the graph, which does not result in DDR graph.

Remark 1.
If G is a DDR graph of diameter three and regularity two, then given a rotation of any length, the graph requires atmost four rotations to regain its original structure. In [13] it was shown that C_7 and C_6 are the only two DDR graphs with diameter three and regularity two. Given a rotation on either of the graphs results in a induced cycle followed by a path. We shall consider them case by case.

Case (i): In C_6, a rotation results in a induced cycle followed by a path, where the length of the induced cycle varies from a minimum of three to a maximum length of $(n - 1)$. Thus depending on the length of the induced cycle formed the graph requires three or four rotations to regain the original structure in forming a C_6.

Case(ii): Similarly for C_7, we again perform three to four rotations to get back C_7 , given a edge rotation of any length.

Theorem 2.2. Given a edge rotation on a cycle, C_n of any length, it requires at most four rotations to again form a cycle.

Proof. Consider a cycle C_n of any length. Applying the definition of edge rotation, lets consider any edge ‘e’ of the cycle for rotation.

Remark 2. Given a rotation of any length on a path, it requires atmost $(n - 1)$ rotations to again form a path of the same length.

4. RANDOMLY EULERIAN FROM ZERO VERTICES

Definition 3. A closed trail containing all vertices and edges is called an Eulerian trail. A graph having Eulerian trail is called an Eulerian graph.

Definition 4. [28] Randomly Eulerian from zero vertices.
Note that all eulerian graphs on $n \leq 5$ vertices are randomly Eulerian from atleast one of their vertices. If $n \geq 6$ is even then take $C_{n/2}$ (n/2 here indicates that we consider half length of the cycle) and use the remaining n/2 vertices to form triangles each with a base on a different edge of the cycle. If $n \geq 7$ is odd, form the graph described above for $n - 1$ and then subdivide any one of its edges with an additional vertex.

Theorem 3.1. If G is a randomly Eulerian graph from 0(zero) vertices, then given a single edge rotation, the resultant graph requires at most four rotations to again form the same graph.

Proof. As defined above if n is even, the first half of n is the inner cycle. Given any rotation of an edge of the inner cycle, we find that it requires at most four rotations to form the cycle as mentioned in Theorem 2.2 and Remark 1.

Theorem 3.2. A randomly Eulerian graph from zero vertices is a ERDG.

Proof. To prove this theorem I shall follow the method proposed

by Chartrand et al. in [7] with slight modifications adaptable to prove the proposed theorem.
If n is even, then for the first n/2 (inner cycle), the construction is as follows:
For $n \geq 3$, let $C : x_1, x_2, \ldots, x_{2n+2}, x_1$ be a $(2n + 2)$ cycle, for $i = 1, 2, \ldots, n$. Let $F_i = C + x_1 x_{i+2}$. For $i = 1, 2, \ldots, n - 1$, define $H_i = F_i \cup F_{i+1}$ and define $H_{n/2} = F_n \cup F_1$. Then $C_{n/2} \cong D_r(H_1, H_2, \ldots, H_{n/2})$. The modification that is done here, is by adding a new vertex v_i for all the first $C_{n/2}$. That is v_i is adjacent to x_i. Next step is to generate the outer cycle for $n \geq 3$. The above specified method shall be used again. Thus, we get $C_{n/2} \cong D_r(H_{n/2+1}, H_{n/2+2}, \ldots, H_n)$. Here also we add a new vertex y_i in the reverse(backward) manner such that y_i is adjacent to x_{2n+2}. Since an edge rotation changes the degree of exactly two vertices, $d_r(H_i, H_j) \geq 1$ for integers i and j such that $1 \leq i \leq j \leq n$. On the other hand,
$d_r(H_1, H_2, \ldots, H_{n/2}),(H_{n/2+1}, H_{n/2+2}, \ldots, H_n), (H_1,H_{n/2+1},H_2,H_{n/2+2},\ldots, H_{n/2},H_n) \cong$ Eulerian graph from Zero vertices.

Example: For generating a Eulerian graph from zero vertices.

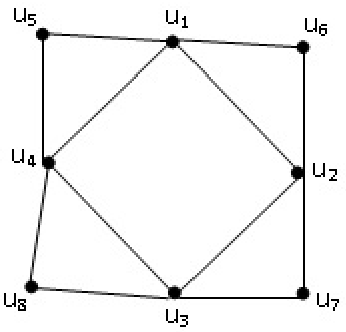

FIGURE 1. Eulerian graph from zero vertices

5. PLANARITY AND ROTATIONS

Here in this section, the discussion would be about the graphs that remain planar even after a single edge rotation. That is, I check out for the graphs that will be planar after an edge rotation. To consider general graphs and finding the planarity of the graph post rotation seems pretty. Hence, consider paths and then consider single edge rotations on them. Upon this then consider the power of such graphs and can later check for the planarity of the graph. As mentioned above in the beginning the denotion G^r denotes the newly obtained graph after a single edge rotation.

Theorem 4.1. The graph $(G^r)^2$ is planar only if G is a path of length 3 or length 4.

Proof. Let the length of the path be less than three. Consider a P_3 as shown below. The complement has just one edge, and applying the definition of rotation we find the resultant is again a P_3. Now taking the first power we obtain a C_3, which is planar which is a trivial case.

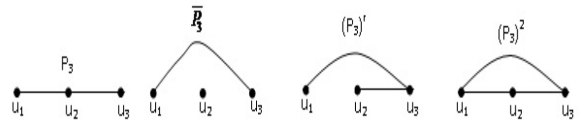

Fig 2 : A planar graph post rotation and power

Now if we consider the length of the path to be greater than five, i.e., P_6 and shall perform the edge rotation on the pendant edges of the path and later take the first power of the resultant graph, which results in a non-planar graph. Thus, by considering the edges in the interior region also results in a non-planar graph. Hence let us consider the length of the path to be three, that is P_4. Here in this graph, the maximal sub graph is of length two along with a pendant edge. Taking the first power of this newly graph, we find that the resultant graph is planar. Similarly the same operation is performed on a P_5. Thus we again find that, the resultant graph is planar.

6. ALGORITHM

In the following algorithm,I shall generate all rotation distance graphs at distance one.

Algorithm 5.1. This algorithm gives the maximum number of graphs that can be obtained by one edge rotation.

Step 1: Input the adjacency matrix of the given graph.
Step 2: In each row , for every removal of 1, one by one, append 0 to 1 , except v_{ii} [diagonal entries].
Step 3: For each row, sum the number of zeros appended to 1.
Step 4: The sum of each rows numbers of zeros appended to 1 gives the number of edge rotated graphs from the given graph.
Step 5: Stop.

Here are few conjectures presented below, that is left as an open problem to the readers which may be of some interest.

Conjecture 5.1. A regular graph is a ERDG.

Conjecture 5.2. A self-centered regular graph is a ERDG.

Remark 3. If G is a ERDG, then the prism of G is again a ERDG.

Conjecture 5.3. A Eulerian graph is a ERDG.

7. ACKNOWLEDGMENTS

I would like to thank the experts who gave their prompt suggestions for the betterment of the article and also contributed towards development of the template.

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