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Lowest Supply and Demand Method to Find Basic Feasible Solution of Transportation Problems

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ABSTRACT
The Transportation Problem (TP) is a particular subclass of a linear programming problem. Economic and social activities are fostered by TP. In order to reduce the transportation cost (TC), we have demonstrated a novel method in this study for obtaining the basic feasible solution (BFS) to transportation problems. "Lowest Supply and Demand (LSD) Method to Find Initial Basic Feasible Solutions of Transportation Problems" is the name of the suggested methodology. Many other strategies have been discussed for solving transportation problems, including Vogel's approximation method (VAM), row minima method (RMM), column minima method (CMM), north-west corner rule (NWCR), and matrix minima method (MMM). Numerical examples are used to compare the suggested method with the well-established method that is currently in use. Compared to the current methods, the suggested method finds the IFS to a transportation problem more quickly. The suggested approach is a compelling way to solve the problem. The investigation's LSD method appears to be simpler than other approaches, requiring less iteration to arrive at a feasible result.

General Terms
Transportation Problem, Minimum Cost, Source, Destination, Stepping Stone Method (SSM), Modified Distribution Method (MODI).

Keywords
Initial Feasible Solution (IFS), Least Supply Demand (LSD), Optimal Solution, Transportation Problem (TP), Vogel Approximation Method (VAM).

1. INTRODUCTION
Transportation problems are a unique LPP subcategory [1, 2]. Because of their widespread real-life applications, transportation problems are popular in operations research [3]. The primary goal of the transportation problem is to reduce the cost of distributing goods to several locations from multiple sources [4]. The amounts that are needed at the demand sites and the amounts that are available at the supply centers define the parameters of the transportation problem. Globalization requires transportation [5]. Tradition, culture, and the supply and demand for products and materials bind all nations together. One big challenge is the problem of material and goods transportation. Operation research makes use of transportation-related issues. It is helpful to provide an optimal answer in mathematics [6].

In today's volatile economic circumstances, discrepancies between the cost of production and the sale price have created significant risks in the consumer market. Day by day, the costs at which everyday goods are sold rise to their limit [7]. The authority is having a challenging time dealing with this disparity. Any manufacturing item's transportation cost rises if the transportation cost of its raw materials rises. An efficient transportation network can have an impact on this industry by lowering the cost of production and the sale price of daily commodities. A manufacturing company's profit can also be increased by reducing transportation costs. The goal of this research is to meet the demand for commodities and their destinations while spending the least amount of money on transportation [8].

The transportation problem solution process consists of three steps [9, 10] such as

- Step 1: Mathematical description of the problem.
- Step 2: Obtaining an initial, basic, feasible solution.
- Step 3: Checking the optimality of the basic feasible solution.

The goal of our research in step 2 is to discover a new way of obtaining the initial fundamental possible solution**.**

2. MATERIALS AND METHODS
2.1 Transportation Problem Model
Transportation problems are one type of network optimization challenge. It transfers products and services from various supply centers to various demand locations, with the goal of minimizing total transportation costs [11].

The goal is achieved if the following conditions are met:

1. Every demand center gets its requirement.
2. The capacity that is available is not exceeded by supply center distributions.

This purpose is accomplished subject to availability and necessity constraints.

2.2 Mathematical Model

The transportation problems states that the total constraints are equal to m + n where m is the total sources and n is the total destinations. Every i^{th} source has a supply capability of s_i, d_j is the requirement demand of each j^{th} destination, and the cost from the i^{th} source to the j^{th} destination is c_{ij}. The purpose of solving the transportation problem is to determine the total transportation cost so that the amount of product x_{ij} that will be transferred from the i^{th} source to the j^{th} destination in order to minimize overall transportation costs [12]. For m sources and n destinations the transportation problem is written as mathematically,

Minimize, $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}\, c_{ij}$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots\ldots\ldots\ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1, 2, \ldots\ldots\ldots\ldots, n$$

and $x_{ij} \geq 0$ for every i and j [13].

3. ALGORITHMS OF TRANSPORTATION PROBLEM

A transportation problem can be addressed with several approaches. There is a common sequence in every approach. The following is a typical order to find an initial basic feasible solution [14].

- Step 1: Establish the objective function and then build the problem within the constraints and non-negativity restrictions. The capacity of supply and demand at each origin and destination serves as a constraint, while the overall cost of transportation serves as the objective function [15].
- Step 2: Use one of the following strategies to arrive at a feasible solution:
  1. Vogel's Approximation Method (VAM)
  2. Row Minima Method (RMM),
  3. Column Minima Method (CMM)
  4. North-West Corner Rule (NWCR)
  5. Matrix Minima Method (MMM)

The following requirements must be met by the solution produced using any of the above-mentioned methods:.

1. The solution has to be feasible, meaning it has to meet all demand and supply restrictions.
2. Total positive allocations must be equal to m + n – 1, where m and n represent the number of rows and columns, respectively [9].

- Step 3: Take one of the following approaches to check that the present initial basic feasible is optimal.
  1. Stepping Stone Method (SSM)
  2. Modified Distribution Method (MODI)

If the present solution is optimal, stop; if not, come up with a new solution.

Step 4: Continue from Step 3 until the optimal value is found.

4. PROPOSED ALGORITHM FOR THE LOWEST SUPPLY DEMAND METHOD

The Lowest Supply and Demand (LSD) method algorithms are given below:

- Step 1: Generate the transportation matrix by using the provided transportation problem.
- Step 2: Identify the smallest supply from the supply column and the smallest demand from the demand row.
- Step 3: Identify the corresponding cell for the smallest supply and smallest demand. Choose the winner in a tie if the smallest supply and smallest demand appear two or more times.
- Step 4: Cross out the satisfied column or row and make changes to the supply and demand. Removal of the row from the transportation table indicates that the supply has been exhausted; similarly, removal of the column from the transportation table indicates that the demand has been met.
- Step 5: Continue doing steps 2 through 4 until all rim criteria are met.

5. NUMERICAL EXAMPLE

Solve a transportation problem using the Lowest Supply Demand (LSD) approach, and then compare the outcome with those of the other approaches now in use.

5.1 Example 1:

With four markets and three warehouses, consider a transportation problem. While the warehouse can accommodate 6, 5, and 4 people, the market demands are 7, 5, 2, and 1. The transportation cost is shown in units by the cell entries.

Table 1. Data of example 1

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2	5	7	3	6
O_2	1	1	4	6	5
O_3	8	2	2	1	4
Demand	7	5	2	1	15

In the transportation table, the total supply (6+5+4=15) is 15 units and the total demand (7+5+2+1=15) is 15 units. The provided transportation problem is a balanced transportation problem because the total supply and the entire demand are equal.

Table 2. Iteration 1

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2	5	7	3	6
O_2	1	1	4	6	5
O_3	8	2	2	1 $\boxed{1}$	4-1=3
Demand	7	5	2		15

In the supply center the smallest supply is 4 units and from the demand the smallest demand is 1 unit. The corresponding cell of the smallest supply and the smallest demand is (3, 4). So, the first allocation is made in the cell (3, 4). The maximum feasible amount is x_{34} = min {4, 1} = 1 unit. It satisfies the requirement of the demand.

Table 3. Iteration 2

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2	5	7	3	6
O_2	1	1	4	6	5
O_3	8	2	2 $\boxed{2}$	1 $\boxed{1}$	3-2=1
Demand	7	5			15

Now the minimum supply is 3 unit and the minimum demand is 2 units and the corresponding cell of the smallest supply and smallest demand is (3, 3). Thus, cell (3, 3) is where the second allocation is made. The maximum feasible amount is x_{33} = min {2, 3} = 2 units. It satisfies the requirement of the demand.

Table 4. Iteration 3

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2	5	7	3	6
O_2	1	1	4	6	5
O_3	8	2 $\boxed{1}$	2 $\boxed{2}$	1 $\boxed{1}$	
Demand	7	5-1=4			15

The minimum supply and the minimum demand except third & fourth column and third row of the reduced transportation table are 1 unit and 5 units respectively.). Thus, cell (3, 2) is where the second allocation is made. The maximum feasible amount is x_{32} = min {1, 5} = 1 units. Therefore, it exhausts the availability of the supply.

Table 5. Iteration 4

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2	5	7	3	6
O_2	1	1 $\boxed{4}$	4	6	5-4=1
O_3	8	2 $\boxed{1}$	2 $\boxed{2}$	1 $\boxed{1}$	
Demand	7				15

The minimum supply and the minimum demand except third and fourth column and third row of the reduced transportation table are 4 units and 5 units respectively. Thus, cell (2, 2) is where the second allocation is made. The maximum feasible amount is x_{22} = min {4, 5} = 4. It satisfies the requirement of the demand.

Table 6. Iteration 5

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2	5	7	3	6
O_2	1 $\boxed{1}$	1 $\boxed{4}$	4	6	
O_3	8	2 $\boxed{1}$	2 $\boxed{2}$	1 $\boxed{1}$	
Demand	7-1=6				15

Now the minimum supply and the minimum demand of the reduced transportation table are 1 and 7 units respectively. The corresponding cell of the smallest supply and the smallest demand is at (2, 1). The cell (2, 1) makes the fifth allocation. The maximum feasible amount is x_{21} = min {1, 7} = 1 unit. Therefore, it exhausts the availability of the supply.

Table 7. Iteration 6

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	2 $\boxed{6}$	5	7	3	
O_2	1 $\boxed{1}$	1 $\boxed{4}$	4	6	
O_3	8	2 $\boxed{1}$	2 $\boxed{2}$	1 $\boxed{1}$	
Demand					15

The cell (1, 1) contains the final allocation. The availability of the supply and the requirement of the demand are equal in this cell. A solution to the transportation problem has been obtained as a result of the fulfillment of all the requirements.

In the transportation table, there are six occupied cells with m rows and n columns and that m + n – 1 = 3 + 4 – 1 = 6 is equal. That is to say, the solution is a basic feasible solution one.

Here, $C_{11} = 2$, $C_{21} = 1$, $C_{23} = 1$, $C_{32} = 2$, $C_{33} = 2$, $C_{34} = 1$

And $X_{11} = 6$, $X_{21} = 1$, $X_{23} = 4$, $X_{32} = 2$, $X_{33} = 2$, $X_{34} = 1$

According to the aforementioned itinerary, the transportation cost is provided by:
Min Z = $C_{11} X_{11} + C_{21} X_{21} + C_{23} X_{23} + C_{32} X_{32} + C_{33} X_{33} + C_{34} X_{34}$
= 2*6 + 1*1 + 1*4 + 2*1 + 2*2 + 1*1
= 12 + 1+ 4 + 2 + 4 + 1
= 24

5.2 Example 2:

Think about a transportation issue where there are three warehouses and four markets. The warehouse has a capacity of 14, 16, and 5, while the market demands are 6, 10, 15, and 14. The unit cost of transportation is represented by the cell entries.

Table 8. Data of example 2

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	35

In the table that the total supply (14+16+5=35) is 35 units and the total demand (6+10+15+4=35) is 35 units. The provided transportation problem is a balanced transportation problem because the total supply and the entire demand are equal.

Table 9. Iteration 1

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2 $\boxed{4}$	5-4=1
Demand	6	10	15		35

In the supply center the smallest supply is 5 units and from the demand the smallest demand is 4 units. The corresponding cell of the smallest supply and the smallest demand is (3, 4). So, the first allocation is made in the cell (3, 4). The maximum feasible amount is x_{34} = min {5, 4} = 4 units. It satisfies the requirement of the demand.

Table 10. Iteration 2

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4 $\boxed{1}$	3	6	2 $\boxed{4}$	
Demand	6 – 1 = 5	10	15		35

Now the minimum supply is 1 unit and the minimum demand is 6 units and the corresponding cell of the smallest supply and smallest demand is (3, 1). Thus, cell (3, 1) is where the second allocation is made. The maximum feasible amount is x_{31} = min {1, 6} = 1 unit. It exhausts the availability of the supply.

Table 11. Iteration 3

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6 $\boxed{5}$	4	1	5	14-5 = 9
O_2	8	9	2	7	16
O_3	4 $\boxed{1}$	3	6	2 $\boxed{4}$	
Demand		10	15		35

The minimum supply and the minimum demand except fourth column and third row of the reduced transportation table are 14 units and 5 units respectively. Thus, cell (1, 1) is where the third allocation is made. The maximum feasible amount is x_{11} = min {14, 5} = 5 units. So, it satisfies the requirement of the demand.

Table 12. Iteration 4

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6 $\boxed{5}$	4 $\boxed{9}$	1	5	
O_2	8	9	2	7	16
O_3	4 $\boxed{1}$	3	6	2 $\boxed{4}$	
Demand		10-9 = 1	15		35

The minimum supply and the minimum demand except first and fourth column and also third row of the reduced transportation table are 9 units and 10 units respectively. Thus, cell (2, 1) is where the fourth allocation is made. The maximum feasible amount is x_{21} = min {9, 10} = 9. It exhausts the availability of the supply.

Table 13. Iteration 5

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6 [5]	4 [9]	1	5	
O_2	8	9 [1]	2	7	16 -1 = 15
O_3	4 [1]	3	6	2 [4]	
Demand			15		35

Now the minimum supply and the minimum demand of the reduced transportation table are 16 and 1 units respectively. The corresponding cell of the smallest supply and the smallest demand at (2, 2). Thus, cell (2, 2) is where the fifth allocation is made. The maximum feasible amount is x_{22} = min {16, 1} = 1 unit. It satisfies the requirement of the demand.

Table 14. Iteration 6

Origin	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	6 [5]	4 [9]	1	5	
O_2	8	9 [1]	2 [15]	7	
O_3	4 [1]	3	6	2 [4]	
Demand					35

The cell (2, 3) contains the final allocation. The availability of the supply and the requirement of the demand are equal in this cell. A solution to the problem has been established as a result of the fulfillment of all the requirements.

In the transportation table, there are six occupied cells with m rows and n columns and that m + n – 1 = 3 + 4 – 1 = 6 is equal. That is to say, the solution is a basic feasible solution one.

Here, $C_{11} = 6$, $C_{12} = 4$, $C_{22} = 9$, $C_{23} = 2$, $C_{31} = 4$, $C_{34} = 2$
And $X_{11} = 5$, $X_{12} = 9$, $X_{22} = 1$, $X_{23} = 15$, $X_{31} = 1$, $X_{34} = 4$
According to the aforementioned itinerary, the transportation cost is provided by:

$$\text{Min } Z = C_{11} X_{11} + C_{12} X_{12} + C_{22} X_{22} + C_{23} X_{23} + C_{31} X_{31} + C_{34} X_{34}$$
$$= 6*5 + 4*9 + 9*1 + 2*15 + 4*1 + 2*4$$
$$= 30 + 36 + 9 + 30 + 4 + 8$$
$$= 117$$

6. RESULTS AND DISCUSSION

The minimum transportation cost of the numerical example 1, is 24. From the result, it is evident that the Lowest Supply and Demand (LSD) method provides the feasible solution is 24, which is equal to VAM, NWCM and RMM and actually equal to the optimal solution. In this example, the proposed method gives a better solution than the Column Minima method and the Matrix Minima method. On the other hand, the minimum transportation cost of the numerical example 2 is 114. From the result, it is evident that the Lowest Supply-Demand (LSD) method provides the feasible solution is 117, which is close to the solution of VAM. In the example-2, the proposed approaches give a better solution than the Column Minima method, Matrix Minima method, Row Minima method and North-West Corner method. In the above two examples, it clear that the Lowest Supply and Demand (LSD) method provides a better solution than other methods except Vogel's approximation method. Sometimes the Lowest supply and demand gives the same result as Vogel's approximation solution. However, the number of iterations of the proposed LSD method is less than Vogel's approximation method. The comparison of the lowest supply and demand method with the existing method is given below:

Table 15. Comparison between existing methods and the proposed method

Transportation Problem	Optimal Solution	NWC	RMM	CMM	LCM	VAM	LSDM
Cij: {(6,4,1,5); (8,9,2,7); (4,3,6,2)} S: (14,16,5) D: (6,10,15,4)	114	128	152	121	156	114	117
Cij: {(2,5,7,3); (1,1,4,6); (8,2,2,1)} S: (6,5,4) D: (7,5,2,1)	24	24	24	39	40	24	24

7. CONCLUSION

A transportation model's principal objective is to provide an effective way to send goods to customers from the source with the minimum cost. In this paper, a new and unique approach has been established to find a transportation problem's basic feasible solution, named the "Lowest Supply-Demand Method." The approach's accuracy has been confirmed through numerical problem solving. This method is easier than other methods because it requires a minimum iteration and takes less time to reach the optimal solution. The solution obtained through the LSD method is either the same as or nearly identical to the optimal solution. However, sometimes it provides a less feasible solution than the VAM method, and this is the limitation of the proposed method. For future studies, the researchers need to write a program code for the suggested method to make computing easier.

8. CONFLICT OF INTEREST

The authors agree that this research was conducted in the absence of any self-benefiting commercial or financial conflicts and declare the absence of conflicting interests with the funders.

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