A New Approach to Solve the Classical Symmetric Traveling Salesman Problem by Highest Suffix Method

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ABSTRACT
This paper presents Highest Suffix method for solving the classical symmetric traveling salesman problem. This concept is an alternative method for solving Traveling Salesman problem (TSP). It is possible to further improve a TSP tour that cannot be improved by other local search methods. To test the performance of the proposed method, two examples are solved here. This is a new approach to solve the classical symmetric travelling salesman problem by highest suffix method. So, this paper shows that the proposed algorithm is efficient for solving the Traveling Salesman problem (TSP).

Keywords
Local search, Symmetric Traveling Salesman Problem, Highest Suffix method, Cost-Constrained Traveling Salesman Problem (CCTSP).

1 INTRODUCTION
Traveling salesman problem is one of the classical challenging combinatorial optimization problems. The objective of the TSP is to minimize the total distance traveled by visiting all the nodes once and only once and then returning to the depot node. The classical formulation of the TSP is stated as follows. Let a network $G = (N, A)$ be defined with N denoting the set of nodes on the network, A denoting the set of arcs and $D = [D_{ij}]$ denoting the matrix of distance. Starting from specific node, the salesman has to visit $N-1$ nodes exactly once and return to the specific node while minimizing the travelled distance. The nodes in the network may represent locations (cites, game parks, mountains, airport sand countries, among others). Network arcs represent links (roads, railway lines and rivers, among others).

A common application of the TSP is the movement of people, equipment and vehicles around tours of duty to minimize the total traveling distance and cost. For example, Post routing is one of the applications of the TSP. The postman problem is modeled as traversing a given set of streets in a city, rather than visiting a set of specified locations. The Cost-Constrained Traveling Salesman Problem is another variant. In the TSP, the goal is to find a tour of a given set of destinations such that the total cost of the tour is minimized. Moreover, the TSP plays an important role in general cost problem, where the houses or streets are far away from each other. If it is possible to create a shortage tour then cost also minimizes. Besides the above mentioned applications, some other seemly unrelated problems are solved by formulating them as the TSP. By considering markers as cities, a genome sequence can be viewed as a TSP path traveling through each marker once and only once. The drilling problem is another application of the TSP with the objective of minimizing the total travel time of the drill. The applications of the TSP are not limited to the examples described above. A detailed review of the applications of the TSP can be found in [1-4]. It has been proved that TSP is NP-hard in which imply that a polynomial bounded exact algorithm for TSP is unlikely to exist.

In this paper, a Highest Suffix method for solving the TSP is presented and its performance is illustrated based on the standard TSP instances. By doing local search using the Generalized Crossing (GC) method, which is developed in [9] for the vehicle routing problem (VRP), each block is explored intensively in order to improve the existing solution. This paper is organized as follows. In 2, a literature review is given on the TSP exact and approximate algorithms.

2 LITERATURE REVIEW
On concept of the Seven Bridges of Königsberg problem, TSP can be traced back to Euler's 18th-century which laid the foundation for graph theory and combinatorial optimization. In the mid-20th century, Karl Menger and Merrill Flood formulate TSP. TSP is a classic NP-hard problem in which a salesman finds out the shortest possible tour that visits a set of fixed locations exactly once and returns to the starting location.

The problem has various practical applications such as transportation, manufacturing fields, various goods supply etc. The TSP has been studied intensively during the last 100 years and many exact and heuristic algorithms have been developed. [9-13]. These algorithms include construction algorithms, iterative improvement algorithms, branch-and-bound and branch-and-cut exact algorithms, and many meta heuristic algorithms such as simulated annealing (SA), tabu search (TS), ant colony (AC) [4] and genetic algorithm (GA). Some of the well-known tour construction procedures are the nearest neighbor procedure by Rosenkrantz et al. [20], the Clarke and Wright savings' algorithm, the insertion procedures, the partitioning approach by Karp [14] and the minimal spanning tree approach by Christofides etc. [5-8] [16-21], The branch exchange is perhaps the best-known iterative improvement algorithm for the TSP. The 2-opt and 3-optheuristics were described in Lin. Lin and Kernighan made a great improvement in quality of tours that can be obtained by heuristic methods. Even today, their algorithm remains the key ingredient in the most successful approaches for finding high-quality tours and is widely used to generate initial solutions for other algorithms. One of the earliest exact algorithms is due to Dantzig et al., in which linear programming (LP) relaxation is used to solve the integer formulation by adding suitably chosen linear inequality to the list of constraints continuously Branch and bound (B&B) algorithms are widely used to solve the TSPs. Several authors have proposed (B&B) algorithm based on assignment problem (AP) relaxation of the original TSP formulation.[21-22] These authors include Eastman, Held and Karp, Smith et al., Carpaneto and Toth , Balas and Crhistofides. Some branch and cut (B&C) based exact algorithms were developed by Crowder and Padberg, Padberg and Hong, Grotschel and Holland. Besides the above mentioned exact and heuristic algorithms, metaheuristic algorithms have been applied successfully to the TSP by a number of researchers. [23-27]. Someone introduced a new method known as zero suffix method to solve TSPs.[5] Some work based on mobile robots' technology was reported by BrummitB and Stentz [15].

SA algorithms for the TSP were developed by Bonomi and Lutton [22], Golden and Skiscim [23] and Nahar et al. [24], Lo and Hus [25] etc. Tabu search metaheuristic algorithms for the TSP have been proposed by Knox [26], and Fiechter [27] etc. The AC is a relative new metaheuristic algorithm which is applied successfully to solve the TSP., Gomez and Banan [29], Bullnheimer et al. [30] and Tsai et al. [31]. Genetic algorithms for the TSP were reported by Grefenstette et al. [32], Whitley et al. [33], and Nguyen et al. [34-40].

2.1 The Highest Suffix Method

Now introducing a new method called the Highest Suffix method for finding an optimal solution to the traveling salesman problem. The Highest Suffix method proceeds as follows.

Step-1: Form the given problem or given network construct the distance symmetric matrix.

Step-2: Find the highest distance component c_{ij} from all the distance components and select the *i*-th row and *j*-th column along which distance component c_{ij} appears. Along this selected cell find the minimum distance component c_{ik} (or c_{kj}) from all the distance components of *i*-th row and *j*-th column. First assignment is to be made in this cell. Choose the minimum distance cell arbitrarily in case of tie for selecting minimum distance component**.**

Step-3: (i) If the selected minimum distance component is c_{ik} in the *i*-th row, find the maximum distance component c_{pk} from the cells appears along the *k*-th column. Now select the minimum distance component c_{pr} from all the distance components of *p*-th row.

(ii) If the selected minimum distance component is c_{kj} in the *j*-th column, find the maximum distance component c_{kp} from the cells appears along the *k*-th row. Now select the minimum distance component c_{rp} from all the distance components of *p*-th column.

Step-4: Continue this process of this step until two assignments are left. If the selected minimum distance component is c_{kj} , Reverse the process described in Step-3. Continue this process of this step until two assignments is left.

Step-5: Omits all the rows and column those are assigning in step-3. After reduce rows and column the matrix select next two cells according to the diagonal position which couple create a tour.

Step 6: According to the selected cells create a initial tour.

2.2 Optimization Process

Check the TSP condition now, if the condition is satisfied then optimal solution is obtained else go to the following steps.

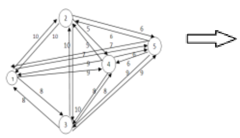
(a) Break the highest path between the two nodes of the circle. Now make a travel from the previous node of the breaking path with the rest of the other nodes which has a small unit in the last table. Continue this process until a complete travel is made. Calculate the total travel cost.

(b) Again reverse the circle and break the highest path between the two nodes of the circle, and then do the same as discussed in step (a). Continue this process until a complete travel is made, and then calculate the total travel cost.

Compare the travel costs between the steps in 3(a) and in 3(b) and the minimum travel cost, thus obtained, is the optimal solution

3 Example -1

Step 1: According to the given problem construct the cost / distance matrix.



	x_1	x_2	x_3	x_4	x_5
x_1	-	**10**	**8**	**9**	**7**
x_2	**10**	-	**10**	**5**	**6**
x_3	**8**	**10**	-	**8**	**9**
x_4	**9**	**5**	**8**	-	**6**
x_5	**7**	**6**	**9**	**6**	-

Fig: Cost/Distance assigned Network Problem 1 Symmetrical Matrix from

Step 2: Comparing all PFIs select HPFI. Here the value of HPFI is 10 which is appears in the (1,2)th ,(2,1)th ,(2,3)th & (3,2)th cell. Along 2nd row & 2nd column select minimum cost / distance value is 5. But here arise a tie. So arbitrarily we select, (2,4) th cell.

	x_1	x_2	x_3	x_4	x_5
x_1	-	**10**	**8**		**7**
x_2		-			
x_3	**8**	**10**	-		**9**
x_4	**9**	**5**	**8**	-	**6**
x_5	**7**	**6**	**9**		-

For Along the selected (2,4) th cell maximum cost is 9 which appears in the (1,4) th cell. Along this cell minimum cost is 7. So, select (1,5) th cell.

	x_1	x_2	x_3	x_4	x_5
x_1	-				
x_2		-			
x_3	**8**	**10**	-		**9**
x_4	**9**	**5**	**8**	-	
x_5	**7**	**6**	**9**		-

Along (1,5) th cell maximum cost is 9 which appears in the (3,5)th cell. Along (3,5)th cell minimum cost is 8 which appears in the cell (3,1)th cell.

	x_1	x_2	x_3	x_4	x_5
x_1	-				
x_2		-			
x_3			-		
x_4		**5**	**8**	-	
x_5		**6**	**9**		-

Step 5: Selected paths: **2 → 4** ;**1 → 5**; **3 → 1** ;5→2;4→3

	x_1	x_2	x_3	x_4	x_5
x_1	-				**7**
x_2		-		**5**	
x_3	**8**		-		
x_4			**8**	-	
x_5		**6**			-

Tour: 1→5→2→ 4→3→1

Cost: 7+ 6 + 5 + 8 + 8 = 34

	x_1	x_2	x_3	x_4	x_5
x_1	-	**10**	**8**	**9**	**7**
x_2	**10**	-	**10**	**5**	**6**
x_3	**8**	**10**	-	**8**	**9**
x_4	**9**	**5**	**8**	-	**6**
x_5	**7**	**6**	**9**	**6**	-

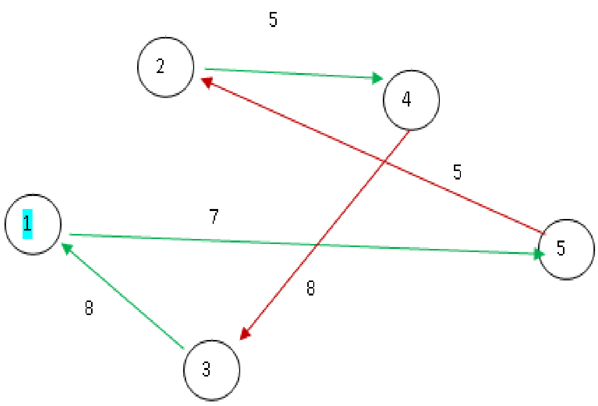


Fig: 1

3.1 Example -2:

According to the problem the symmetrical matrix is:

	x_1	x_2	x_3	x_4	x_5	x_6
1	-	8	5	3	1	2
2	8	-	4	9	2	8
3	5	4	-	9	6	7
4	3	9	9	-	1	1
5	1	2	6	1	-	9
6	2	8	7	1	9	-

Accordingly, our proposed algorithm the selected paths are 4-5;6-4;3-2;1-6;2-1;5-4
Here create three sub-tours. For eliminate sub-tours we can select the following paths according to the guideline of proposed algorithm: 4-5-3-2-1-6-4 / 1+6+4+8+1+1=21

4 → 5 → 3 → 2 → 1 → 6 → 4

Total distance = 1+6+4+8+1+1 = 21

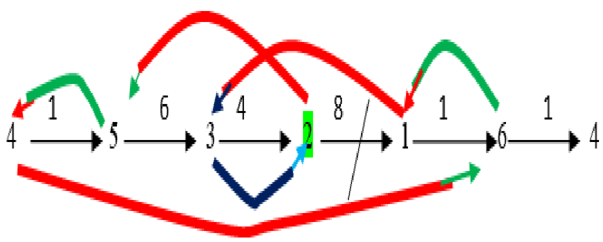


Fig 2

Final selected paths: 2→ 5 ; 1→ 3 ; 3→ 2 ; 5→ 4 ; 6→ 1 ; 4→ 6 .

Tour: 2 → 5 → 4 → 6 → 1 → 3 → 2

Distance: 2 + 1 + 1 + 2 + 5 + 4 = 15

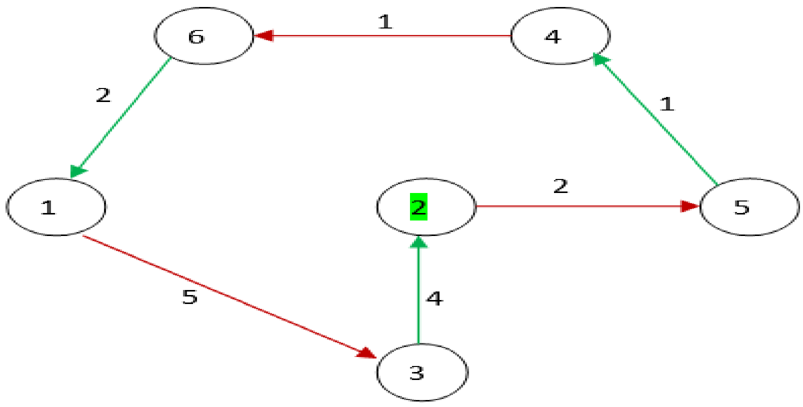


Fig 3

4. CONCLUSIONS

This paper presents a Highest Suffix method to solve TSPs. the proposed method is able to obtain better solutions for TSPs when compared with a co-adaptive neural network method proposed in the literature. We believe that the performance of our method can be further improved by hybridizing with metaheuristic algorithms, such as tabu search and ant colony optimization, and so this is an area of further research. In addition, the method to the TSP as described in this paper can be extended to other similar or related combinational optimization problems as well, such as the vehicle routing problems and machine scheduling problems.

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