Ranking Decision making units using Fuzzy Multi-Objective Approach

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ABSTRACT
This paper presents Data Envelopment Analysis (DEA) has been used in a wide variety of applied research and it is a linear programming methodology that has been widely used to evaluate the performance of a set of decision-making units (DMUs). It requires crisp input and output data. However, in reality input and output cannot be measured in a precise manner. Firstly using DEA to evaluate the efficient and inefficient decision-making units (DMUs) with the (CCR) model. Secondly the resulted weights for each input and output are considered as fuzzy sets and are then converted to fuzzy number. Thirdly using Fuzzy multi-objective approach to find the highest and lowest of the weighted values. Fourthly usingthe results from stage it to rank from highest to lowest. An application from banking industry is presented.

General Terms
Data Envelopment Analysis, Fuzzy multi-objective.

Keywords
Data envelopment analysis; ranking methods in DEA; Multi-objective data envelopment analysis; Fuzzy data envelopment analysis; Fuzzy multi-objective approach.

1. INTRODUCTION
Data Envelopment Analysis (DEA) is a powerful tool for assessing the performance of organizations and their functional units Majid et al.[24]. DEA spans the boundaries of several academic areas and receive increasing importance as a tool for evaluating and improving the performance of manufacturing and service operations.DEA is a non-parametric technique for measuring the relative efficiencies of a set of organizations with which consume multiple-inputs to produce multiple-outputs. The organizations are called the decision-making units, or DMUs. The main idea is to evaluate the relative efficiency of a set of homogenous DMUs by using a ratio of the weighted sum of outputs to the weighted sum of inputs. It generalizes the usual efficiency measurement from a single-input, single-output ratio to a multiple-input, multiple-output ratio. This technique was originally introduced by Farrell [14] and popularized by Charnes el al.[3] The CCR model considers only constant returns to scale while. Banker et. al. [15] BCC model work under assumption of variable returns to scale. Although DEA is a powerful tool for efficiency measurement, there are some limitations that have to be considered. One important limitation involves the sensitivity of the DEA to the data. Because DEA is a methodology focused on frontiers or boundaries, "noise", or errors from data measurement can cause significant problems. Therefore, to successfully apply DEA, one has to have accurate measurement of both the inputs and outputs. However, in some situations, such as in a manufacturing system, a production process or a service system, inputs and outputs are volatile and complex. Adel Hatami-Marbini et al. [5]used the TOPSIS (technique for order preference by similarity to the ideal solution) with DEA for measuring quantitative performance it is integrated into a four phase fuzzy DEA framework to measure the efficiencies of a set of DMUs and rank them with fuzzy input–output levels. Neto et al. [12] used interval DEA frontier in situations where one input or output is subject to uncertainty in its measurement and is presented as an interval data. They built an efficient frontier without any assumption about the probability distribution function of the imprecise variable. They take into account only the minimum and the maximum values of each imprecise variable. Gharib and Jahromi [2] Used classical Data Envelopment Analysis (DEA) models with fuzzy concept to determined different weights of factors and they presented a model for eliminating the weaknesses. and they assigns each DMU weights to factors in a way to maximize its efficiency. DEA has been applied in many situations such as: health care (hospitals, doctors), education (schools, universities).banks. manufacturing, benchmarking, management evaluation, fast food restaurants, and retail stores.

The rest of this paper is organized as follows: Section 2, Review of ranking methods in the DEA. Section 3, Multi-objective Data Envelopment Analysis. Section 4, Fuzzy Data Envelopment Analysis. Section 5, Fuzzy multi-objective approach. Section 6, An application from banking industry is introduced and conclusion is drawn in Section7.

2. REVIEW OF RANKING METHODS IN THE DEA
Data Envelopment Analysis (DEA) was first proposed by Charnes et al. [3] and is a non-parametric method of efficiency analysis for comparing units relative to their best peers (efficient frontier). Mathematically, DEA is a linear programming-based methodology for evaluating the relative efficiency of a set of decision making units (DMUs) with multi-inputs and multi-outputs. DEA evaluates the efficiency of each DMU relative to an estimated production possibility frontier determined by all DMUs. The advantage of using DEA is that it does not require any assumption on the shape of the frontier surface and it makes no assumptions concerning the internal operations of a DMU.

Let us assume that *n* consume varying amounts of *m* different inputs to produce *s* different outputs. Assume that x_{ij} (i=1,2…..,m) is quantity of input and y_{rj} (r=1,2…..,s) is quantity of output r produced by DMU_j.The CCR model for DMU_P is then written as:

$$Max\ \theta_p = \sum_{r=1}^{s} u_r y_{rp}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ip} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} v_i x_{ij} \leq 0 \quad \forall j, \qquad (1)$$

$$u_r, v_i \geq 0 \forall\ r, i, p$$

Note that the DMUs with $\theta_p^* = 1$, is called the efficient unit, and those units with $\theta_p^* \neq 1$ are called inefficient units. where v_i and u_r in model (1) are the input and output weights assigned to the *ith* input and *rth* output. The DEA approach used to deal with inaccurate inputs and outputs. Aigner et al. [9] proposed a stochastic frontier model to deal with the variation of data in DEA models. However, this model cannot be used for the multiple-input and multiple-output case. Other researchers have investigated the sensitivity analysis of DEA to the addition or removal of inputs and outputs. Most of these studies have simply used simulation techniques like the one in Banker et al. [15]. Furthermore, these methods have shortcomings and they also cannot deal with the linguistic input and output there are several approaches in the literature to rank both efficient, as well as inefficient, DMUs in DEA. Adler et al. [1] classified these approaches into six streams .Majid et al. [25]Perhaps super-efficiency is the most well known, most widely applied and researched ranking method inDEA. This approach was pioneered by Andersen et al. [4].The model reformulated the standard DEA linear programs (LP) by omitting the corresponding column in the technological matrix. However, omitting a DMU from the corresponding matrix might cause some technical problems such as infeasibility. another stream mentioned in Adler et al. [1] involves the use of multivariate statistics in the DEA which alternative approach suggested in the literature involves the use of statistical techniques in alliance with DEA to achieve a complete rankingone of the major aims of the methodologies described in this section is to close the gap between DEA and the classical statistical approaches. The use of multi-criteria decision-making (MCDM) methods to rank DMUs is another stream that was mentioned in Adler et al. [1] the MCDM literature was entirely separate from DEA research, when Golany[11] combined interactive, multiple-objective linear programming and DEA. Whilst the MCDM literature does not consider a complete ranking as their ultimate aim, they do discuss the use of preference information to further refine the discriminatory power of the DEA models. In this manner, the decision-makers could specify which inputs and outputs should lend greater importance to the model solution. Other streams that were mentioned in Adler et al. [1] this is approaches based on benchmarking, pioneered by Torgersen et al. [16] achieved a complete ranking of efficient DMUs by measuring their importance as a benchmark for inefficient DMUs which were developed by Bardhan et al. [8] cross-efficiency approach is another stream that was classified [1] pioneered by Sexton et al.[17] the approach evaluates the performance of a DMU with respect to the optimal input and output weights of other DMUsa limitation in using this approach is that the factor weights obtained from the DEA models may not be unique. The existence of an alternative optimal solution in efficiency evaluation of DMUs causes some difficulties and some techniques have been proposed to obtain robust factor weights for use in the construction of the cross-efficiencies method.

3. MULTI-OBJECTIVE DATA ENVELOPMENT ANALYSIS

Malekmohammadi et al. [19] established the relations between the output-oriented dual DEA model and the minimax reference point formulations, namely the super-ideal point model, the ideal point model and the shortest distance model. Through these models, the decision makers' preferences are considered by interactive trade-off analysis procedures in multiple objective linear programming. Makui et al. [20] used a multiple objective linear programming (MOLP) approach for generating a common set of weights in the DEA and they proposed a method to generate a common set of weights for all DMUs which are able to produce a vector of efficiency scores closest to the efficiency scores calculated from the standard DEA and it provide a common base for ranking the DMUs. Yang et al. [21] used interactive MOLP methods were investigated to conduct efficiency analysis and set realistic target values in an integrated way with the DMs preferences taken into account and with the DM supported to explore what could be technically achievable. Park et al. [22]used a DEA model and statistical method to formulate a nonlinear multiple objective optimization (MOO) model. The DEA model is applied to a set of the input–output data generated from the existing branches, in order to identify inefficient data and drop them from further consideration. Relatively excessive uses of inputs and/or output shortfalls are reflective of inefficiency.

Multi-objective Linear Programming (MOLP)

Veeramani et al. [23] Multi-objective Linear Programming (MOLP) Problems is an interest area of research, since most real-life Problems have a set of conflict objectives.

A mathematical model of the MOLP problem can be written as follows:

$$\begin{aligned} &Max\ Z_1(x) = C_1x \\ &Max\ Z_2(x) = C_2x \qquad (2) \\ &Max\ Z_3(x) = C_3x \\ &Subject\ to\ x \epsilon\ X = \{x \epsilon E^n / Ax = b\ ,\ x \geq 0\} \end{aligned}$$

Where x isan n – dimensional vector of decision variables.$Z_1(\mathcal{X})$,……,$Z_k(\mathcal{X})$ are k – distinct linear objective function of the decision vector $\mathcal{X}$. C_1, C_2,…C_k are n - dimensional cost factor vectors, A is an *m*x*n* constraint matrix, *b* is an *m* – dimensional constant vector.

4. FUZZY DATA ENVELOPMENT ANALYSIS

The fuzzy CCR models cannot be solved by a standard LP solver like a crisp CCR model because coefficients in the fuzzy CCR model are fuzzy sets Adel Hatami-Marbini et. al [5]with the fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA model need to be clarified and generalized. The corresponding fuzzy linear programming problem is usually solved using some ranking methods for fuzzy sets. Hossainzadeh et. al. [10] classified that have been published on solving fuzzy DEA problems can be categorized into four distinct approaches: tolerance approach, defuzzification approach, α-level based approach, and fuzzy ranking approach. Details can be getfromLertworasirikul et al. [26] They also proposed a new approach based on possibility measure in which constraints considered as fuzzy event. Although possibility measure has been widely used, Liu and Liu [13] presented the concept of credibility measure in Wen et al. [18] extended the Charnes, et al. [3] (CCR) model to a fuzzy DEA model by using credibility measure.For solving the fuzzy model,Each of the proposed approaches to solving fuzzy DEA models has both advantages and shortcomings in the way it treats uncertain data in DEA models. For example, the tolerance approach fuzzifies the inequality or equality signs but it does not treat fuzzy coefficients directly. However, often it is the inputs and outputs that are imprecise. The defuzzification approach is simple but the uncertainty in inputs and outputs (i.e., possible range of values at different α-levels) is effectively ignored. The α-level based approach provides fuzzy efficiency but requires the ranking of fuzzy efficiency sets. The fuzzy ranking approach provides fuzzy efficiency for an evaluated DMU at a specified α-level. In their fuzzy ranking approach, Guo and Tanaka [27] compare fuzzy efficiencies using only one number at a given α-level. This ignores the possible range of fuzzy efficiency at that α-levelin addition, this approach requires solving a bi-level linear programming model.

$$\text{Max}\theta_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp}$$

$$\text{s.t} \quad \sum_{r=1}^{s} v_i \tilde{x}_{ip} = 1 \qquad (3)$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{r=1}^{s} v_i \tilde{x}_{ij} \le 0 \quad J \ne p$$

$$u_r, v_i \ge 0 \forall r, i$$

where "~" indicates the fuzziness. $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are fuzzy inputs and fuzzy outputs, respectively. where u_r and v_i (r =1,….m; i=1,….,h)are the crisp decision variables. In the above model, they assume that the right hand sides of the constraints are crisp value because they are similarto the original CCR model used for normalization of the value of the efficiency in the objective function Adel Hatami-Marbini et al. [25].

5. FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING (FMOLP)

The proposed method for ranking completely all efficient and inefficient, DMUs consists of four stages. Majid et al.[24] In Firstly using DEA to evaluate the efficient and inefficient decision-making units (DMUs) with the CCR model. Secondly the resulted weights for each output are considered as fuzzy sets and are then converted to fuzzy number. Thirdly using fuzzy multi-objective approach to find the highest and lowest of the weighted values. Fourthly using the results from stage thirdly to ranked based on the values of their objective functions.

Firstly 1. In this stage, n aforementioned DMUs with m inputs and s outputs are evaluated using the CCR model.

Secondly 2. Assume that $u^*_{rj} = u^a_{rj}$ (j = 1,2,…..,n and r = 1,2,…., s) and $v^*_{ij} = v^b_{ij}$ (j = 1,2,…..,n and i = 1,2,…., s) are optimal solutions of the input and output weights assigned to the ith input and rth output in the CCR model for efficient and inefficient, DMUs. Suppose that K=$\{(r | u^a_{rj} > 0)\}$ and K^{ι} =$\{(r | u^a_{rj} > 0)\}$Corresponding with each weight, and M=$\{(i | v^b_{ij} > 0)\}$and M'=$\{(i | v^b_{ij} > 0)\}$ Corresponding with each weight, define the membership function for fuzzy numbers $\tilde{u}_{rj}$ (j = 1, 2,…., n) and $\tilde{v}_{ij}$(j = 1, 2,…., n)as follows:

$$\mu_{\tilde{u}_{rj}}(\tilde{u}_{rj}) = \frac{u^a_{rj} - \tilde{u}_{rj}}{u^a_{rj} - u^b_{rj}} \quad r \in \text{k}$$

$$\mu_{\tilde{v}_{ij}}(\tilde{v}_{ij}) = \frac{v^b_{ij} - \tilde{v}_{ij}}{v^b_{ij} - v^c_{ij}} \quad i \in \text{M}$$

Where u^a_{rj} and u^b_{rj}= 0 are the upper and lower bounds of the output weights, v^b_{ij} and v^c_{ij}= 0 are the upper and lower bounds of the input weights resulted from solving Model(1) for DMU$_J$(j = 1, 2,...., n).

Thirdly 3.the following Fuzzy multi-objective approach for each efficient and inefficient DMUs.

$$Z_1(x) = max\mu_{\tilde{u}_{rj}}(\tilde{u}_{rj}) = \frac{U^a_{rj} - \tilde{u}_{rj}}{u^a_{rj}} \quad r \in \text{K}$$

$$Z_2(x) = min\mu_{\tilde{v}_{ij}}(\tilde{v}_{ij}) = \frac{v^b_{ij} - \tilde{v}_{ij}}{v^b_{ij}} \quad i \in \text{M}$$

s. to

$$\tilde{u}_r y_{rj_0} = \sum_{j=1}^{n} y_{rj} \lambda_j \quad \forall_r = 1,2,\dots,s$$

$$\tilde{v}_i x_{ij_0} = \sum_{j=1}^{n} x_{ij} \lambda_j \quad \forall_i = 1,2,\dots,z \qquad (4)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

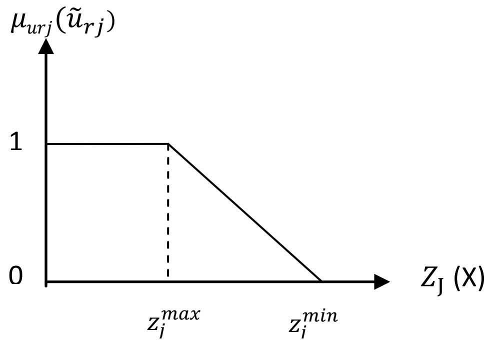
$$y_{rj}, x_{ij} \ge 0,\ \tilde{u}_r \text{Where } r \in \text{k}\ \tilde{v}_i \text{Where } i \in \text{M}$$

Where$\tilde{v}_i$ and $\tilde{u}_r$ in model (4) are the input and output weights assigned to the ith input and rth output.

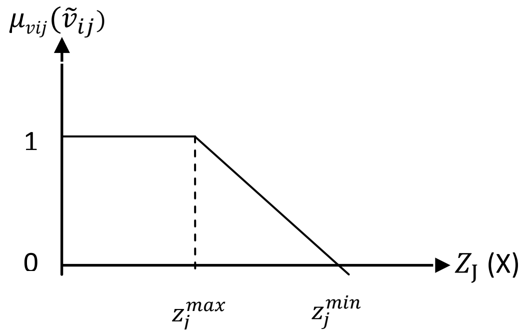
Let min and max for (z_j^{min} and z_j^{max}).
The objective function takes the values between $z_j^{min}$and z_j^{max}.

$$\mu_{vrj}(\tilde{u}_{rj}) = \begin{cases} 1 \\ (z_j(\mathcal{X}) - z_j^{min}) / (z_j^{max} - z_j^{min}) \\ 0 \end{cases} \qquad \begin{matrix} z_j(\mathcal{X}) \ge z_j^{max} \\ z_j^{min} \ge z_j(\mathcal{X}) \ge z_j^{maz} \quad j = 1,2,3 \\ z_j(\mathcal{X}) \le z_j^{min} \end{matrix} \qquad (5)$$

$$\mu_{\tilde{v}_{ij}}(\tilde{v}_{ij}) = \begin{cases} 1 \\ (z_j(\mathcal{X}) - z_j^{min}) / (z_j^{max} - z_j^{min}) \\ 0 \end{cases} \qquad \begin{matrix} z_j(\mathcal{X}) \ge z_j^{max} \\ z_j^{min} \ge z_j(\mathcal{X}) \ge z_j^{maz} \quad j = 1,2,3 \\ z_j(\mathcal{X}) \le z_j^{min} \end{matrix} \qquad (6)$$



Finger 1: Linear membership function



Finger 2: Linear membership function

Where z_j^{max} and z_j^{min} are the upper and lower bounds for $\tilde{u}_{rj}$, respectively and z_j^{max} and z_j^{min} are the upper and lower bounds for $\tilde{v}_{ij}$, respectively, of the jth objective function z_j. The linear membership can be determined by requiring the DM to select the objective values interval$[z_j^{max}, z_j^{min}]$. In practical situations, a possible interval for imprecise Objective values can be estimated based on the experience and knowledge of Decision making (DM) or experts, and their equivalent membership values of the DM in the interval [0,1] finger 1,2 shows the graph of the linear membership functions for equations(5,6)Using the above membership functions we formulate a crisp model by introducing an augmented variableλ as:

$$\begin{aligned} &\textit{Maximize } \lambda \\ &\text{s. to} \\ &\mu_{urj}(\tilde{u}_{rj}) \geq \lambda \qquad r=1,2,\ldots\ldots, k \\ &\mu_{vij}(\tilde{v}_{ij}) \geq \lambda \qquad i=1,2,\ldots\ldots, m \\ &\tilde{u}_r y_{rj_0} = \textstyle\sum_{j=1}^{n} y_{rj}\,\lambda_j \quad \forall_r = 1,2,\ldots\ldots,s \qquad (7) \\ &\tilde{v}_i x_{ij_0} = \textstyle\sum_{j=1}^{n} x_{ij}\,\lambda_j \quad \forall_i = 1,2,\ldots\ldots,z \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ &y_{rj}, x_{ij} \geq 0, \quad 0 \leq \lambda \leq 1 \end{aligned}$$

6. AN APPLICATION

A real data set taken from cofaceegypt (2010 / 2013) in comparing the efficiencies of 10 banks is given in Table 1. The comparison is based on four inputs and three outputs.

Table 1: A set of 10 DMUs with four inputs and three outputs

DMUs	inputs				Outputs		
	I_1	I_2	I_3	I_4	O_1	O_2	O_3
1	21	910	108780	124550	4829502	133987	9543640
2	50	1147	66896	22331	3872243	114162	5811105
3	221	11665	874347	1238432	7515853	1118084	35930730
4	155	4739	973029	190239	26598400	646848	50623700
5	71	1933	481364	35480	5208940	188229	19525659
6	15	871	174670	73546	6423874	581436	7772234
7	61	2530	234244	27931	4960279	406786	5375491
8	93	2268	425216	125916	13224455	347693	30187879
9	429	15322	2106480	3106992	75844804	1133496	194968015
10	140	3700	643084	108983	24856338	119428	40987562

Table 2: Inputs–outputs used for assessing efficiency and non- efficiency.

inputs	Outputs
I_1:Number of Branches	O_1: loans & Overdrafts
I_2:Number of Employes	O_2: Trading Investments
I_3: Admin. & General expenses	O_3: Customer Deposits
I_4: Provisions	

Table 3: Efficiency and Non-Efficiency the Optimal Weights Resulted from CCR Model

DMU_j	$v_{1j}^*=v_{1j}^b$	$v_{2j}^*=v_{2j}^b$	$v_{3j}^*=v_{3j}^b$	$v_{4j}^*=v_{4j}^b$	$u_{1j}^*=u_{1j}^a$	$u_{2j}^*=u_{2j}^a$	$u_{3j}^*=u_{3j}^a$	CCR
1	0.11	0	0.89	0	0.94	0.06	0	1
2	0	0	0.41	0.59	0.60	0.40	0	1
3	0.03	0	0.97	0	0	0.38	0.62	0.558
4	0.66	0	0	0.34	0.08	0.05	0.87	1
5	0	0	0	1	0.36	0	0.64	1
6	0.10	0	0	0.90	0.56	0.44	0	1
7	0	0	0	1	0.76	0.24	0	1
8	0	1	0	0	0.53	0	0.47	1
9	0	0	1	0	0.09	0	0.91	1
10	0	0	0	1	0.56	0	0.44	1

Table 4: The Membership functions

DMU_j	$v_{1j}^*=v_{1j}^b$	$v_{2j}^*=v_{2j}^b$	$v_{3j}^*=v_{3j}^b$	$v_{4j}^*=v_{4j}^b$	$u_{1j}^*=u_{1j}^a$	$u_{2j}^*=u_{2j}^a$	$u_{3j}^*=u_{3j}^a$
1	$\frac{0.11-\tilde{v}_{11}}{0.11}$	0	$\frac{0.89-\tilde{v}_{31}}{0.89}$	0	$\frac{0.94-\tilde{u}_{21}}{0.94}$	$\frac{0.11-\tilde{u}_{21}}{0.11}$	0
2	0	0	$\frac{0.41-\tilde{v}_{32}}{0.41}$	$\frac{0.59-\tilde{v}_{42}}{0.59}$	$\frac{0.60-\tilde{u}_{12}}{0.60}$	$\frac{0.40-\tilde{u}_{22}}{0.40}$	0
3	$\frac{0.03-\tilde{v}_{13}}{0.03}$	0	$\frac{0.97-\tilde{v}_{11}}{0.97}$	0	0	$\frac{0.38-\tilde{u}_{23}}{0.38}$	$\frac{062.-\tilde{u}_{33}}{0.62}$
4	$\frac{0.66-\tilde{v}_{14}}{0.66}$	0	0	$\frac{0.34-\tilde{v}_{44}}{0.34}$	$\frac{0.08-\tilde{u}_{14}}{0.08}$	$\frac{0.05-\tilde{u}_{24}}{0.05}$	$\frac{0.87.-\tilde{u}_{34}}{0.87}$
5	0	0	0	$\frac{1-\tilde{v}_{45}}{1}$	$\frac{036.-\tilde{u}_{15}}{0.36}$	0	$\frac{0.64-\tilde{u}_{35}}{0.64}$
6	$\frac{0.10-\tilde{v}_{16}}{0.10}$	0	0	$\frac{0.90-\tilde{v}_{46}}{0.90}$	$\frac{0.56-\tilde{u}_{16}}{0.56}$	$\frac{0.44.-\tilde{u}_{26}}{0.44}$	0
7	0	0	0	$\frac{1-\tilde{v}_{47}}{1}$	$\frac{0.76-\tilde{u}_{21}}{0.76}$	$\frac{024.-\tilde{u}_{27}}{0.24}$	0
8	0	$\frac{1-\tilde{v}_{28}}{1}$	0	0	$\frac{0.53-\tilde{u}_{18}}{0.53}$	0	$\frac{0.47.-\tilde{u}_{38}}{0.47}$
9	0	0	$\frac{1-\tilde{v}_{39}}{1}$	0	$\frac{0.09-\tilde{u}_{19}}{0.09}$	0	$\frac{0.91-\tilde{u}_{39}}{0.91}$
10	0	0	0	$\frac{1-\tilde{v}_{410}}{1}$	$\frac{0.56-\tilde{u}_{110}}{0.56}$	0	$\frac{0.44-\tilde{u}_{310}}{0.44}$

***DMUs 1*: Fuzzy multi-Objective linear programming for ranking units.**

$$max\, z_2 = \frac{0.94 - \tilde{u}_{11}}{0.94}$$

$$max\, z_2 = \frac{0.06 - \tilde{u}_{21}}{0.06}$$

$$min\, z_2 = \frac{0.11 - \tilde{v}_{11}}{0.11}$$

$$min\, z_2 = \frac{0.89 - \tilde{v}_{31}}{0.89}$$

s.to.

$4829502\tilde{u}_{11}$-4829502λ_1-3872243λ_2-7515853λ_3-26598400$\lambda_4$5208940λ_5-6423874λ_6-4960279λ_7-13224455λ_8-75844804λ_9-24856338λ_{10} = 0
$133987\tilde{u}_{21}$-133987λ_1-114162λ_2-1118084λ_3-646848λ_4-188229λ_5 -581426λ_6-406786λ_7-347693λ_8-1133496λ_9-119428λ_{10} = 0
$9543640\tilde{u}_{31}$-9543640λ_1-5811105λ_2-35930730λ_3-50623700λ_4-19525659λ_5-7772234λ_6-5375491λ_7-30187879λ_8-194968015λ_9-40987562λ_{10} = 0

$21\tilde{v}_{11}$-21 λ_1-50λ_2-221λ_3-155λ_4-71λ_5-15λ_6-61λ_7-93λ_8-429λ_9-140λ_{10}=0

$910\tilde{v}_{21}$-910λ_1-1147λ_2-11665λ_3-4739λ_4-1933λ_5-871λ_6-2530λ_7 -2268$\lambda_8$15322λ_9-3700λ_{10}=0

$108780\tilde{v}_{31}$-108780λ_1-66896λ_2-874347λ_3-973029λ_4-481364λ_5-174670λ_6 -234244λ_7-425216λ_8-2106480λ_9-643084λ_{10}=0

$124550\tilde{v}_{41}$-124550 λ_1-22331λ_2-1238432λ_3-190239λ_4-35480λ_5-73546λ_6 -27931λ_7-125916λ_8-3106992λ_9-108983λ_{10}=0

$\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9+\lambda_{10}=1$

This is done by solving the lower and upper bounds of the optimal values first and Optimal values of these problems are Z_1= 0.15, Z_1= -15.65and Z_2= -13.20 ,Z_2= -140.03and Z_3 = -5.49 ,Z_3= -184.70 andZ_4 = 0.31 , Z_4= -20.69

-Solving Fuzzy multi-Objective linear programming problems:

Max λ

s.to

$0.060271\tilde{u}_{11}-0.9433428\lambda \le 1$

$0.11735\tilde{u}_{21}-0.986016\lambda \le 1$

$0.048693\tilde{v}_{11}-0.994645\lambda \le 1$

$0.0495988\tilde{v}_{31}-0.955828\lambda \le 1$

$4829502\tilde{u}_{11}$-4829502λ_1-3872243λ_2-7515853λ_3-26598400$\lambda_4$5208940λ_5 -6423874λ_6-4960279λ_7-13224455λ_8-75844804λ_9-24856338λ_{10} = 0

$133987\tilde{u}_{21}$-133987λ_1-114162λ_2-1118084λ_3-646848λ_4-188229λ_5 -581426λ_6-406786λ_7-347693λ_8-1133496λ_9-119428λ_{10} = 0

$9543640\tilde{u}_{31}$-9543640λ_1-5811105λ_2-35930730λ_3-50623700λ_4-19525659λ_5-7772234λ_6-5375491λ_7-30187879λ_8-194968015λ_9-40987562λ_{10} = 0

$21\tilde{v}_{11}$-21 λ_1-50λ_2-221λ_3-155λ_4-71λ_5-15λ_6-61λ_7-93λ_8-429λ_9-140λ_{10}=0

$910\tilde{v}_{21}$-910λ_1-1147λ_2-11665λ_3-4739λ_4-1933λ_5-871λ_6-2530λ_7 -2268$\lambda_8$15322λ_9-3700λ_{10}=0

$108780\tilde{v}_{31}$-108780λ_1-66896λ_2-874347λ_3-973029λ_4-481364λ_5-174670λ_6 -234244λ_7-425216λ_8-2106480λ_9-643084λ_{10}=0

$124550\tilde{v}_{41}$-124550 λ_1-22331λ_2-1238432λ_3-190239λ_4-35480λ_5-73546λ_6 -27931λ_7-125916λ_8-3106992λ_9-108983λ_{10}=0

$\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9+\lambda_{10}=1$

Consequently, they obtain the optimal value of Max λ = 0.9217 and optimal value is $\tilde{u}_{11}$=2.1656,$\tilde{u}_{21}$=0.7771.$\tilde{u}_{31}$=2.4650, $\tilde{v}_{11}$=1.7094,$\tilde{v}_{21}$=4.3495,$\tilde{v}_{31} = 2.3995$,$\tilde{v}_{41}$=2.7522and λ_1=0.1370,λ_2=0.1749,λ_3=0.1934,λ_4=0.1077,λ_5=0.0293, λ_6=0.1775,$\lambda_7 = 0.0421$,λ_8=0.0563,λ_9=0.0102,λ_{10}=0.0716

DMU_j	$\tilde{V}^*_{1J}$	$\tilde{V}^*_{2J}$	$\tilde{V}^*_{3J}$	$\tilde{V}^*_{4J}$	$\tilde{U}^*_{1J}$	$\tilde{U}^*_{2J}$	$\tilde{U}^*_{3J}$	Efficiency and non-efficiency score	new rank
1	1.7094	0	2.3995	0	2.1656	0.7771	0	-1.1662	10
2	0	0	0.7173	1.8190	1.3207	0.7152	0	-0.5004	7
3	0.126	0	3.408	0	3.655	0	0.665	-.586	8
4	4.6382	0	0	3.8069	2.9850	1.9666	3.9964	0.5029	5
5	0	0	0	3.199	1.8771	0	2.9483	1.6664	3
6	2.0191	0	0	1.8179	1.3350	1.3522	0	-1.1498	9
7	0	0	0	2.8824	2.8022	1.6828	0	1.6026	4
8	0	2.4067	0	0	2.1652	0	2.3756	2.1341	1
9	0	0	3.1919	0	2.8538	0	2.1744	1.8363	2
10	0	0	0	3.2415	1.7249	0	1.7833	0.2667	6

7. CONCLUSION

In this paper, Data Envelopment Analysis is used to evaluate the relative efficiency of decision-making units. The use of standard DEA methods usually resulted in a number of DMUs being efficient and inefficient. And the resulted weights to define fuzzy number. after that using fuzzy multi-objective approach to find the highest and lowest of the weighted values. finally used the results to rank from highest to lowest.

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